Calculation of Coherent Synchrotron Radiation in General Particle Tracer

T. Miyajima[1], Ivan V. Bazarov[2]
KEK-PF[1], Cornell University[2]

9 July, 2008

CSR in GPT

- 1D CSR wake calculation in GPT using D. Sagan's formula.
  - General Particle Tracer (GPT) is a particle tracking code, which includes 3D space charge effect based on a nonequidistant multigrid Poisson solver or a point-to-point method.
  - The routine can calculate 1D-wake functions for arbitrary beam trajectories as well as CSR shielding effect.
  - In particular, the CSR routine does not assume ultrarelativistic electron beam and is therefore applicable at low beam energies in the injector.

- I. V. Bazarov and T. Miyajima, "Calculation of Coherent Synchrotron Radiation in General Particle Tracer", Proc of EPAC 2008, MOPC024

- D. Sagan, "AN EFFICIENT FORMALISM FOR SIMULATING THE LONGITUDINAL KICK FROM COHERENT SYNCHROTRON RADIATION", Proc of EPAC 2006, THPCH024

Sagan’s formula

Sagan's formula 1
Two particle interaction

- The source particle at point P'.
- An electric field E(P) at the position of the kicked particle at point P and time.
- The Lienard-Wiechert formula

$$\mathbf{E}(P) = \frac{e}{\gamma^2}\frac{\mathbf{L}-L\boldsymbol{\beta}'}{(L-\mathbf{L}\cdot\boldsymbol{\beta}')} + \frac{e}{c^2}\frac{L\times\left[(\mathbf{L}-L\boldsymbol{\beta}')\times\mathbf{a}'\right]}{(\mathbf{L}-L\cdot\boldsymbol{\beta}')^3}$$

- The CSR term

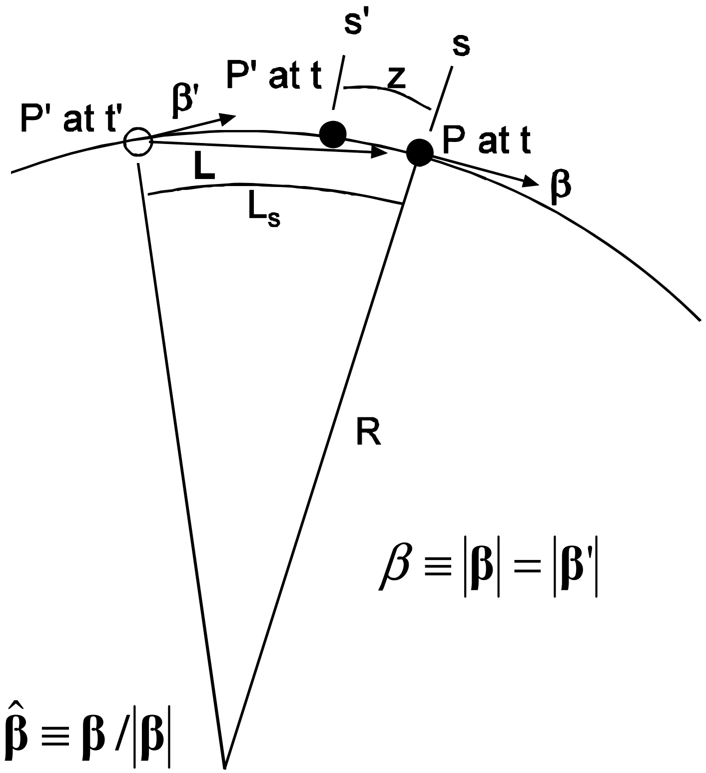
$$\mathbf{E}_{CSR} \equiv \mathbf{E}-\mathbf{E}_{SC}$$

- Here, the space charge term is

$$\mathbf{E}_{SC}(P) \equiv \frac{e\gamma\left[z\hat{\mathbf{s}}+x\hat{\mathbf{x}}+y\hat{\mathbf{y}}\right]}{\left(\gamma^2 z^2+x^2+y^2\right)^{3/2}}$$

- The rate of energy change is given by

$$K_{CSR} = e\hat{\boldsymbol{\beta}}\cdot\mathbf{E}_{CSR} = e\hat{\boldsymbol{\beta}}\cdot\left(\mathbf{E}-\mathbf{E}_{CSR}\right)$$



$$\beta \equiv |\boldsymbol{\beta}| = |\boldsymbol{\beta}'|$$

$$\hat{\boldsymbol{\beta}} \equiv \boldsymbol{\beta}/|\boldsymbol{\beta}|$$

Sagan's formula 2
Space charge term

- The space charge term

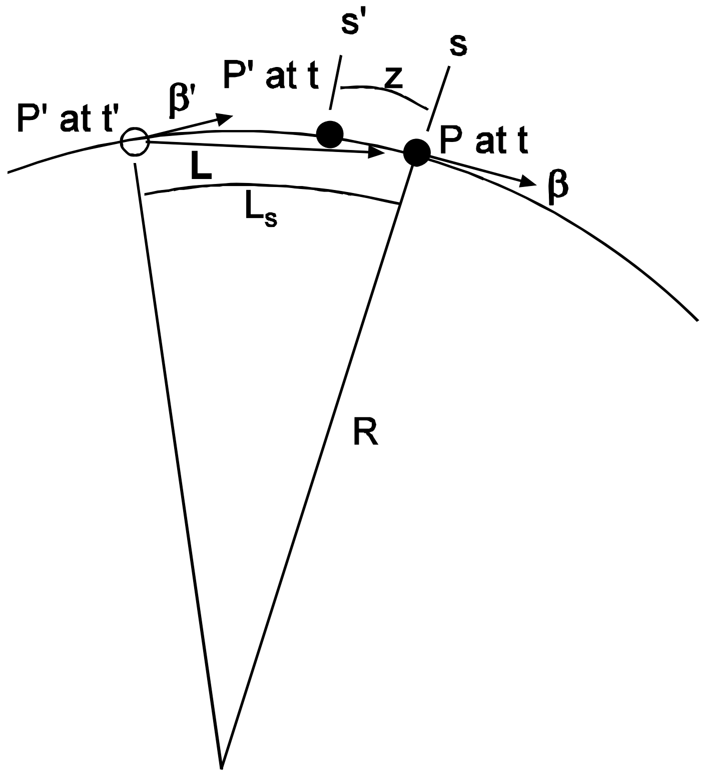
$$\mathbf{E}_{SC}(P) \equiv \frac{e\gamma\left[z\hat{\mathbf{s}} + x\hat{\mathbf{x}} + y\hat{\mathbf{y}}\right]}{\left(\gamma^2 z^2 + x^2 + y^2\right)^{3/2}}$$

- The longitudinal distance is required to calculate the space charge term.
- The change of the longitudinal position of the source particle is

$$L_s - z = \beta c\left(t - t'\right) = \beta L$$

- The longitudinal distance between P' and P is

$$z = L_s - \beta L$$



Sagan's formula 3
Calculation of z on arbitrary orbit

- The orbit is divided into N elements from O.
- The path length: $L_s = d + v_1$ $\qquad v_1 = \sum_{i=1}^{N} d_i$
- v and w components of the vector **L**:

$$L_v = v + R\sin\phi \qquad v = v_1 - v_3 \qquad v_3 = \sum_{i=1}^{N} d_i\left(\frac{1}{2}\psi_i + \frac{1}{2}\psi_i g_i d_i + \frac{1}{6} g_i^2 d_i^2\right)$$

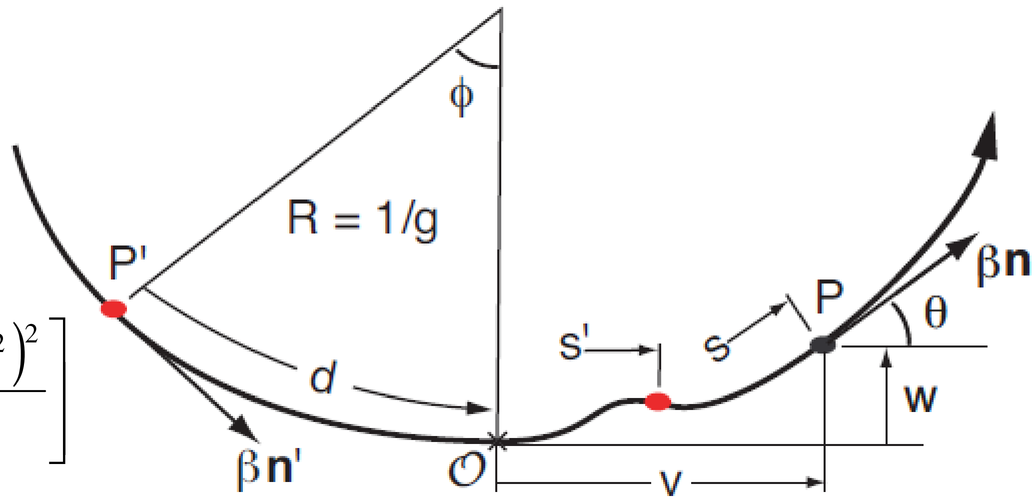
$$L_w = w - R(1-\cos\phi) \qquad w = \omega_2 \qquad \omega_2 = \sum_{i=1}^{N} d_i\left(\psi_i + \frac{1}{2} g_i d_i\right)$$

$$L = \sqrt{L_v^2 + L_w^2}$$

- The distance, z:

$$z = L_s - \beta L$$

$$z = \frac{v_1 + d}{2\gamma^2} + \left[v_3 + \frac{g^2 d^3}{6} - \frac{1}{8}\frac{\left(2\omega_2 - g d^2\right)^2}{v_1 + d}\right]$$



Sagan's formula 4
Calculation of CSR kick on arbitrary orbit

- CSR kick:

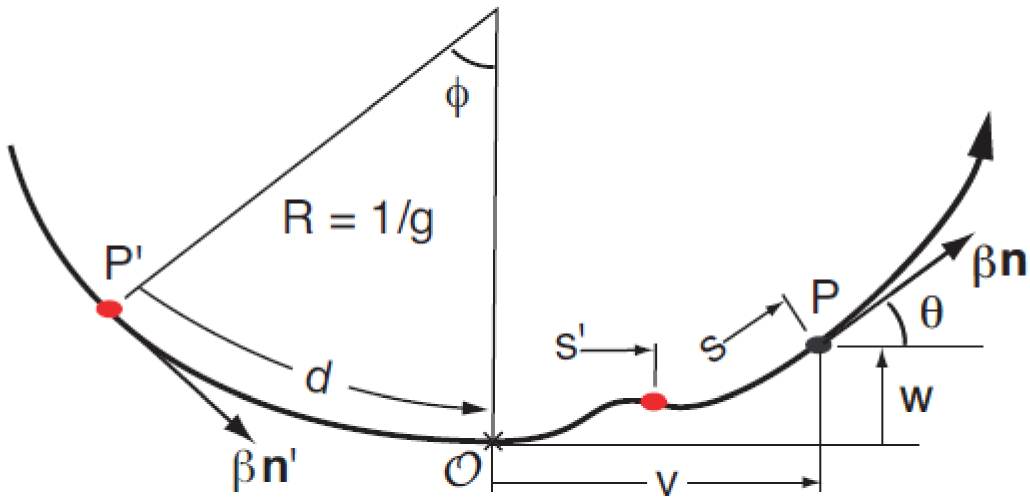
$$K_{CSR} = 4e^2\gamma^4\tau^2\left\{\frac{g(\tau^2-\alpha^2)(\alpha-\tau\kappa)}{(\tau^2+\alpha^2)^3}+\frac{\tau^2-\alpha^2+2\tau\alpha\kappa}{(\tau^2+\alpha^2)^3}\right\}-\frac{e^2}{\gamma^2 z^2}$$

$$\tau = \gamma(d+v_1)$$

$$\alpha = \gamma^2\left(\omega_2 + gdv_1 + \frac{1}{2}gd^2\right)$$

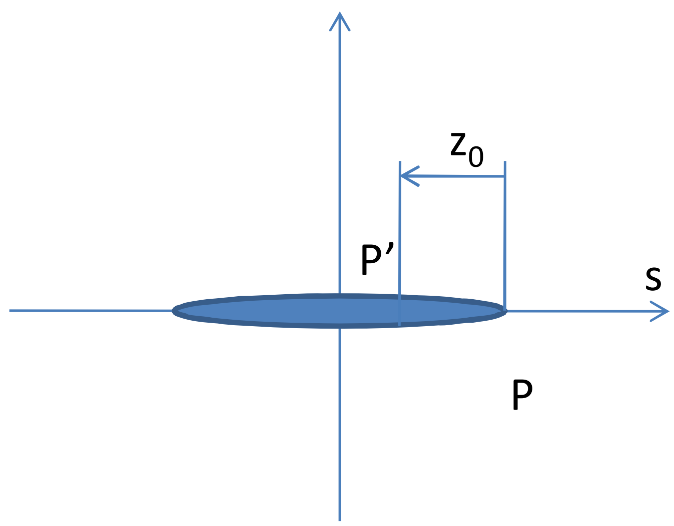
$$\kappa = \gamma(\theta + gd)$$

$$z = \frac{v_1+d}{2\gamma^2}+\left[v_3+\frac{g^2d^3}{6}-\frac{1}{8}\frac{(2\omega_2-gd^2)^2}{v_1+d}\right]$$



Procedure of CSR calculation

1. Calculate i, which satisfy $z_i = z_0$
2. Calculate ν_1, ω_1, g and d with i
3. Calculate K_{CSR} with i
4. Shift z0 and repeat 1 to 3.



CSR calculation in GPT

Commands of GPT/CSR

- Command name
  - csr1Dwakexz();
- Assumption
  - It is assumed that the particles move on x-z plane. Namely, the vertical component of the average velocity is zero.
- Options
  - The GPT/CSR has 16 options.

Options of GPT/CSR

1. CSRTimestep (double) (s)
2. CSRCalcTstep (double) (s)
3. CSRMeshNbin (long)
4. CSRBGTolerance (double)
5. CSRMeshBoxSize (double)
6. CSRMeshNbfac (double)
7. CSRMeshStep (double) (m)
8. CSRTriangleWidth (double) (m)
9. CSRSign (double)
10. CSRHshield (double) (m)
11. CSRNimage (int)
12. CSRDriftLength (double) (m)
13. CSRCalcArea (double) (m)
14. CSRArcRadius (double) (m)
15. CSRArcAngle (double) (rad)
16. CSROutputWake (double) (m)

```
#-----------------------------
# example of CSR calculation
#-----------------------------
csr_dt     = 10.0e-12;
csr_tstep = 0.0;
csr_Nb    = 0;
csr_bgtol  = 1.0e-2;
csr_nstd   = 20.0;
csr_mNbfac = 0.1;
csr_mdl    = 0.06e-3;
csr_dtri   = 0.6e-3;
csr_sign   = -1.0;
csr_h      = 1.0;
csr_Nh     = 0;
csr_inids  = 10.0;
csr_xin    = -10.0;
csr_xout   = 10.0;
csr_zin    = -10.0;
csr_zout   = 10.0;
csr_arcr   = 0.0;
csr_arcang = 0.0;
csr_wfrom  = 0.0;
csr_wto    = 0.0;
csr_wstep  = 0.0;
#------------------------------------------
# please comment out the following line
# for calculation without CSR
#------------------------------------------
csr1Dwakexz("CSRTimestep", csr_dt, "CSRCalcTstep", csr_tstep, "CSRMeshNbin", csr_Nb,
"CSRBGTolerance", csr_bgtol, "CSRMeshBoxSize", csr_nstd, "CSRMeshNbfac", csr_mNbfac,
"CSRMeshStep", csr_mdl, "CSRTriangleWidth", csr_dtri, "CSRSign", csr_sign,
"CSRHshield", csr_h, "CSRNimage", csr_Nh, "CSRDriftLength", csr_inids,
"CSRCalcArea", csr_xin, csr_xout, csr_zin, csr_zout,
"CSRArcRadius", csr_arcr, "CSRArcAngle", csr_arcang,
"CSROutputWake", csr_wfrom, csr_wto, csr_wstep);
```

Energy Loss and Spread (1)

- The steady-state energy loss and spread for various beam energies are compared as calculated by GPT/CSR, elegant, and analytical expression for a circular orbit.

•Bending radius: $\rho = 1.0$ m
•Bunch length: $\sigma_s = 0.6$ mm
•Initial distribution: Gaussian
•Bunch charge: $Q = 80$ pC.

•The CSR routine in elegant includes the assumption of ultrarelativistic beam.
•GPT/CSR reproduces the analytical result accurately.

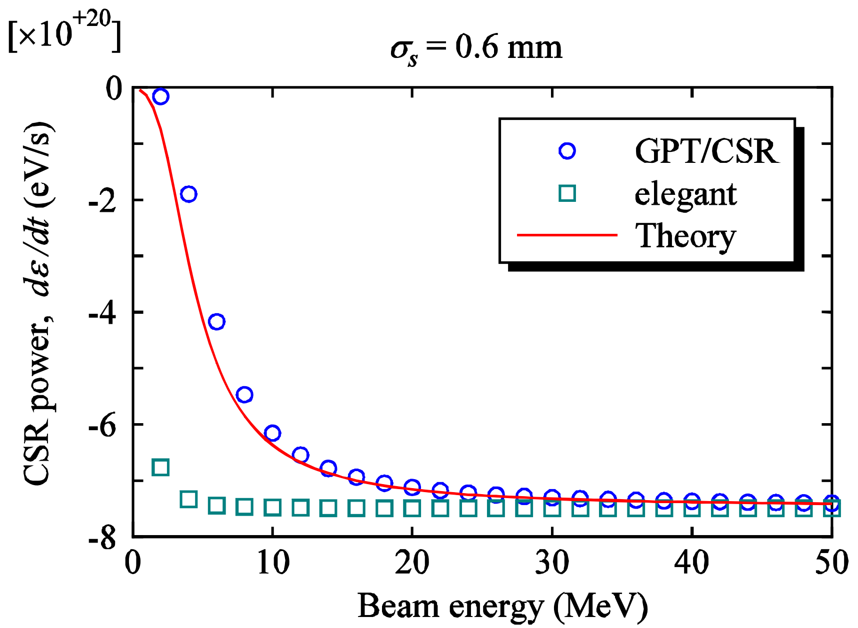
Analytical expression derived by C. Mayes

$$\frac{d\varepsilon}{dt} = -\frac{2}{3}\frac{(r_e m_e c^2)c\beta^4\gamma^4}{\rho^2}N(1+(N-1)T(a))$$

$$a = 3/2 \cdot \gamma 3\sigma_s /(\beta\rho)$$

$$T(a) = \frac{9}{32\pi}\frac{1}{a^3}\left(e^{\frac{1}{8a^2}}\sqrt{\pi}K_{5/6}\left(\frac{1}{8a^2}\right) - 2\pi a\right)$$

$K_{5/6}(x)$: the modified Bessel function
N : the number of election in the bunch
r_e : the classical electron radius



Energy loss and spread (2)

- The results of GPT/CSR and elegant both reproduce well the analytical result for higher beam energy, $E_0 > 40\mathrm{MeV}$.
- The results of elegant and the theory diverge to infinity for $E_0 \to 0$.
- The result of GPT/CSR approaches zero as expected.

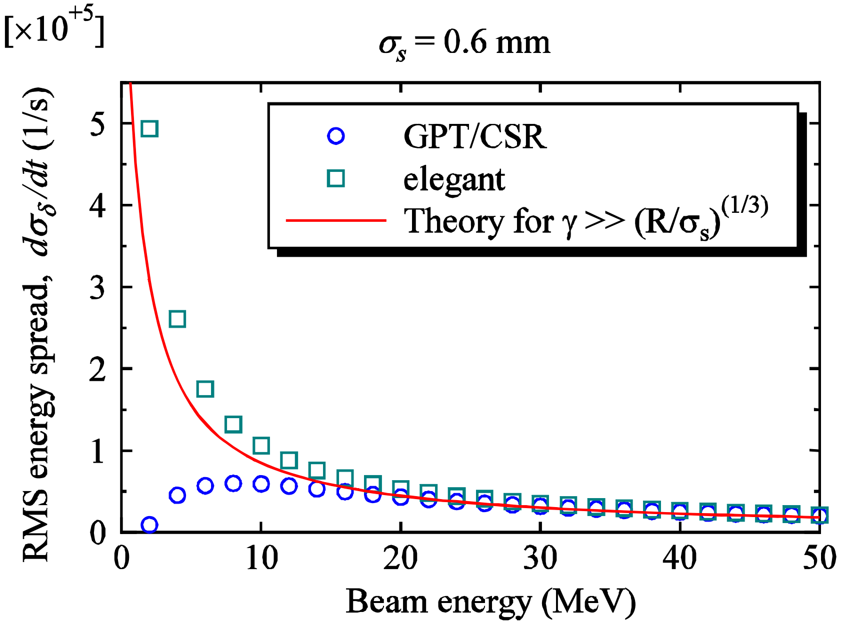
Analytical expression with the assumption of $\gamma >> (r/\sigma_s)^{1/3}$ [1,2]

$$\frac{d\sigma_\delta}{dt} \approx 0.22 \frac{r_e N c \beta}{\gamma \rho^{2/3} \sigma_s^{4/3}}$$

c : the speed of light

γ: the Lorentz energy factor

$$\beta = (1 - 1/\gamma^2)^{1/2}$$



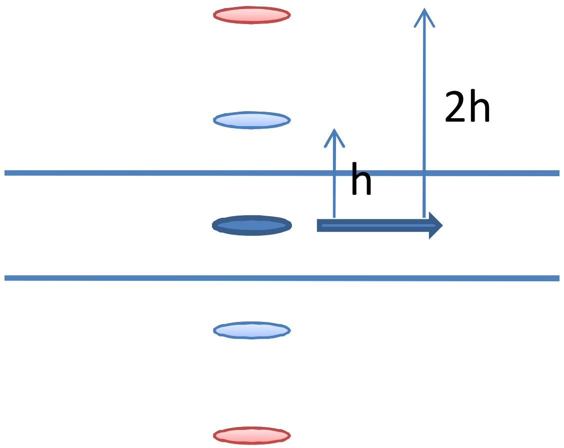
[1] P. Emma and R. Brinkmann, Proceedings of PAC97, Vancouver, B.C., Canada, 1997, pp. 1679-1681.
[2] Ya. S. Derbenev. et.al., TESLA FEL-Report 1995-05.

These results show that the GPT/CSR is effective for wide range of beam energies, and can be used to investigate beam dynamics in ERL and FEL photoinjectors.

CSR shielding effect

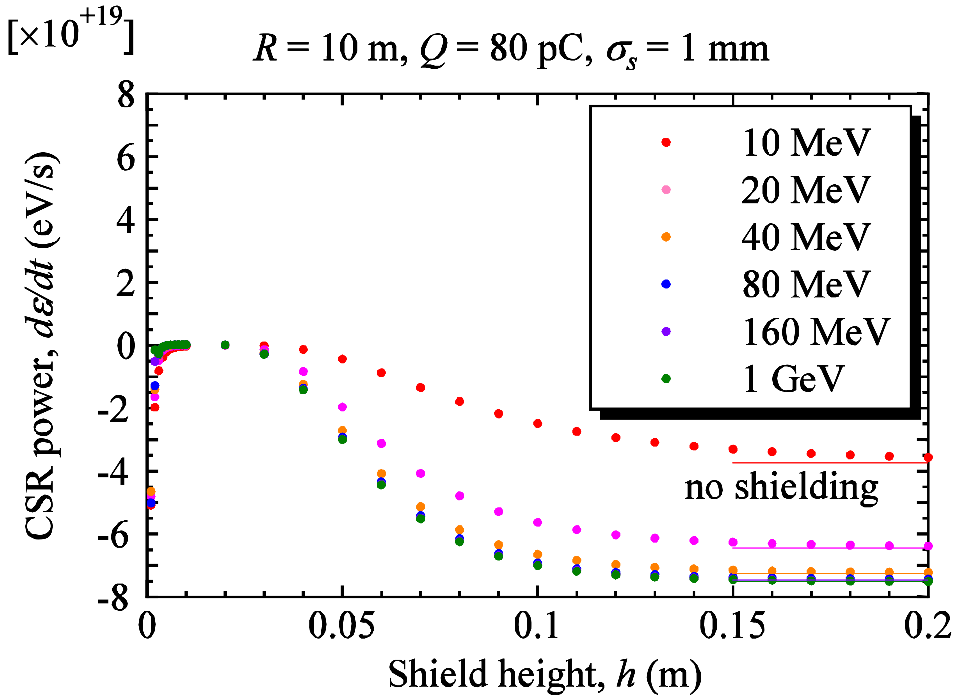
- Image charge layer

Chamber height, h



The effect of CSR shielding is calculated by GPT/CSR for a circular orbit.

- Bending radius: $\rho = 10.0$ m
- Bunch length: $\sigma_s = 1.0$ mm
- Initial distribution: Gaussian
- Bunch charge: $Q = 80$ pC.
- Number of image charge layers: 32

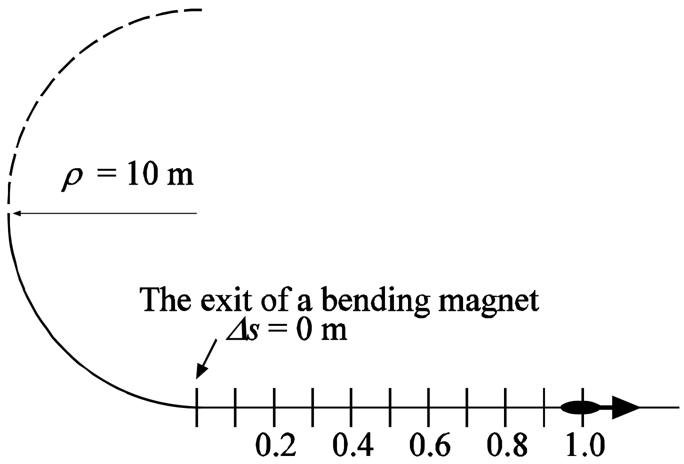


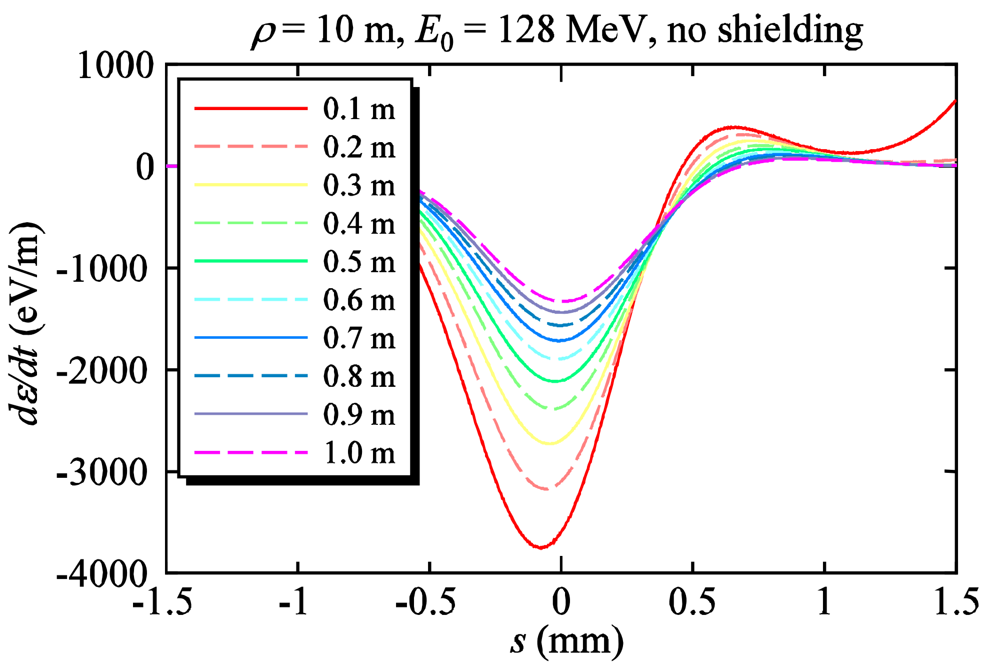
As the shielding height increases, the energy loss approaches to the analytical value.

CSR in transient state without shielding

- As an example of CSR effect in a transient state, the CSR wake form is calculated by GPT/CSR after the exit of a bending magnet.

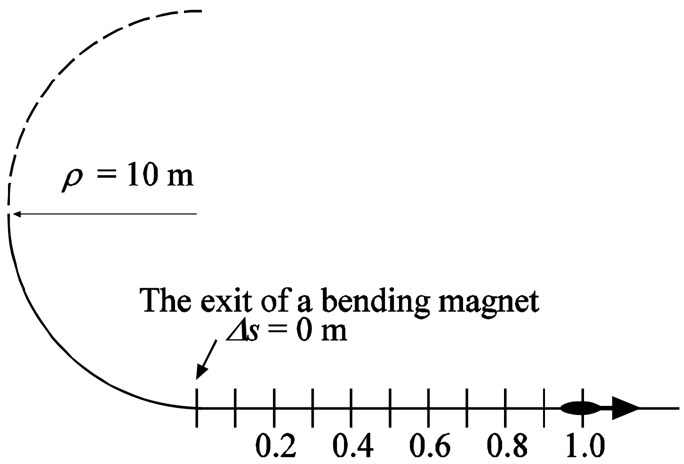
•Beam energy: 128 MeV
•Bending radius: $\rho = 10.0$ m
•Bunch length: $\sigma_s = 0.3$ mm
•Initial distribution: Gaussian
•Bunch charge: $Q = 80$ pC
•Shielding chamber height: $h = \infty$
•Number of image charge layers: 32

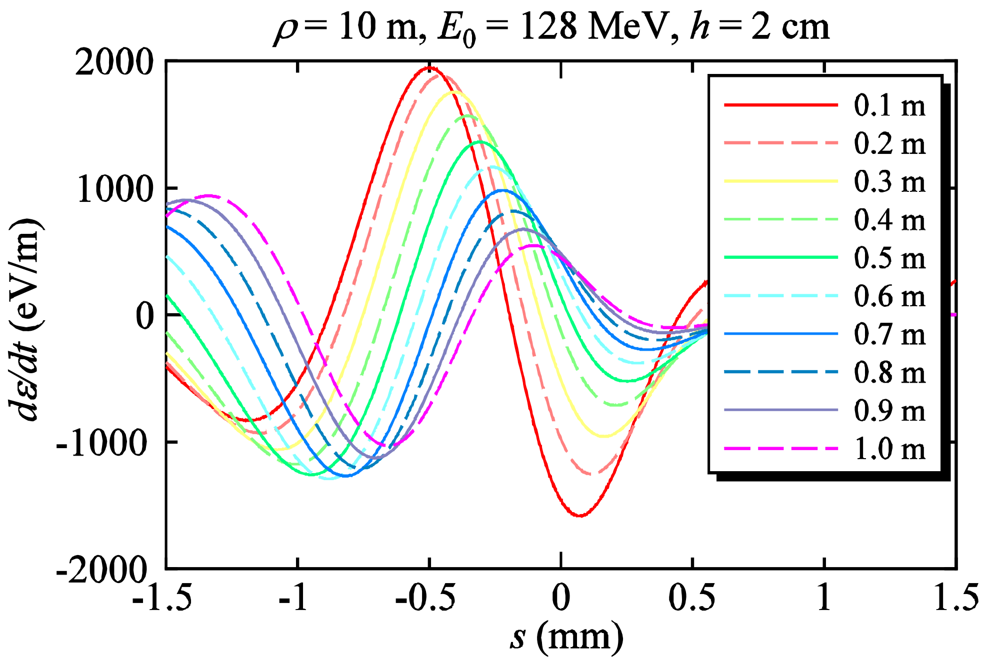




CSR in transient state with shielding

- Beam energy: 128 MeV
- Bending radius: $\rho = 10.0$ m
- Bunch length: $\sigma_s = 0.3$ mm
- Initial distribution: Gaussian
- Bunch charge: $Q = 80$ pC
- Shielding chamber height: $h = 2$ cm
- Number of image charge layers: 32

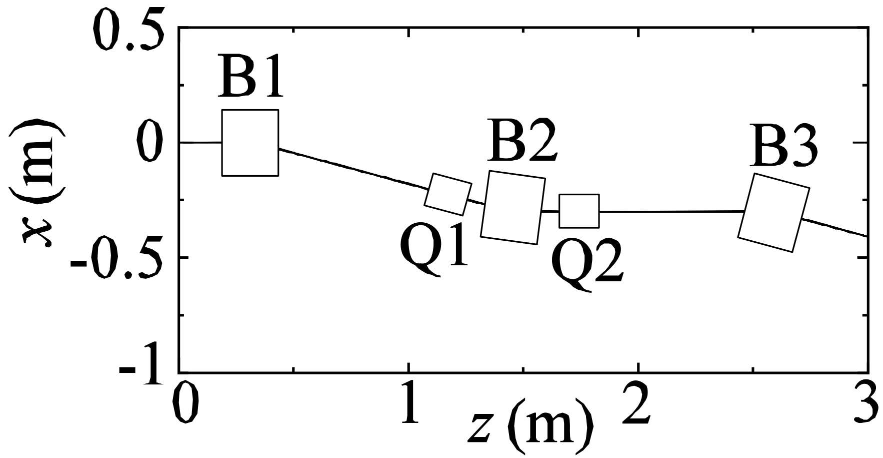




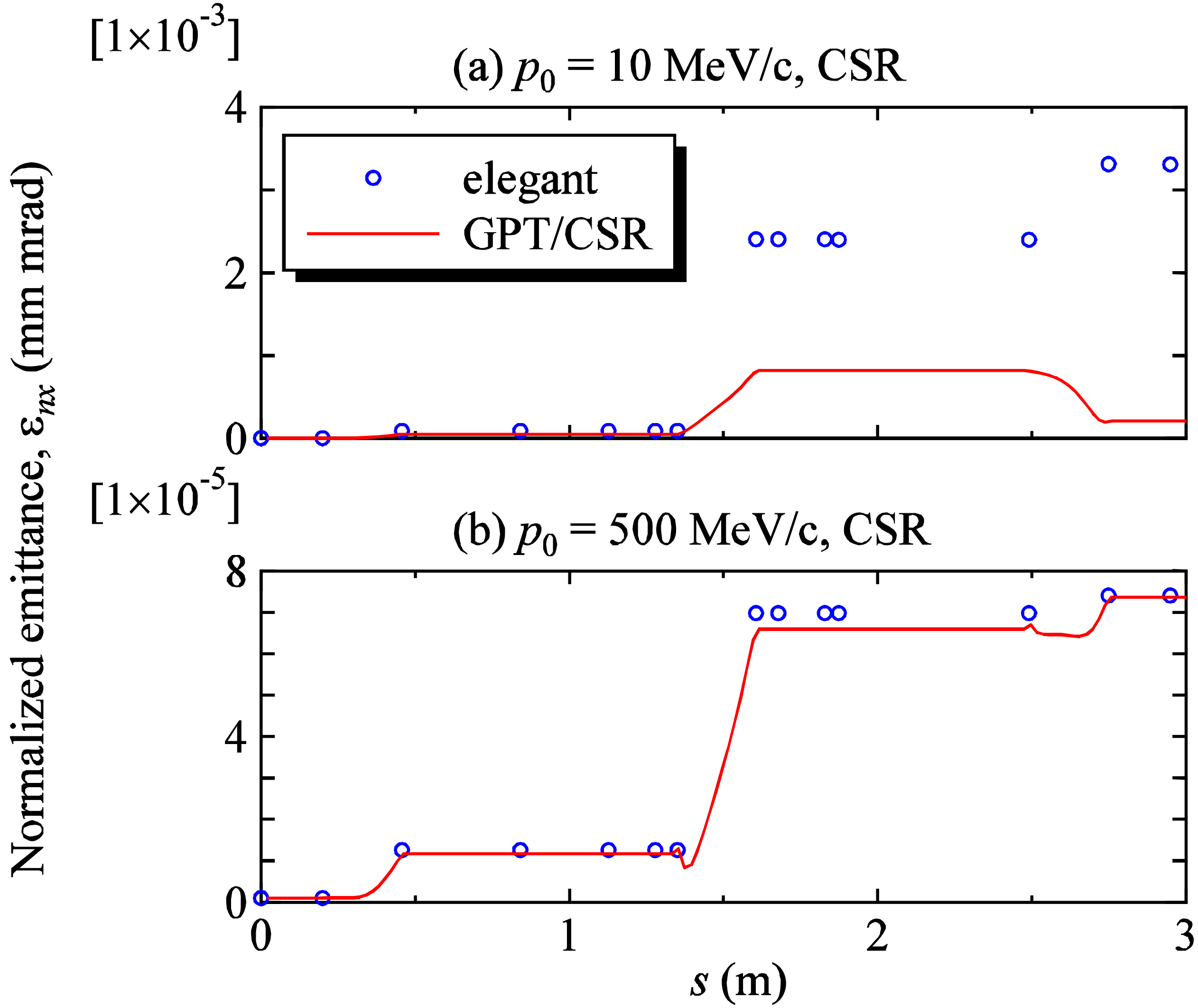
The figures show that the CSR wake reduces as the distance from the exit of the bending magnet increases as expected.

CSR in merger section

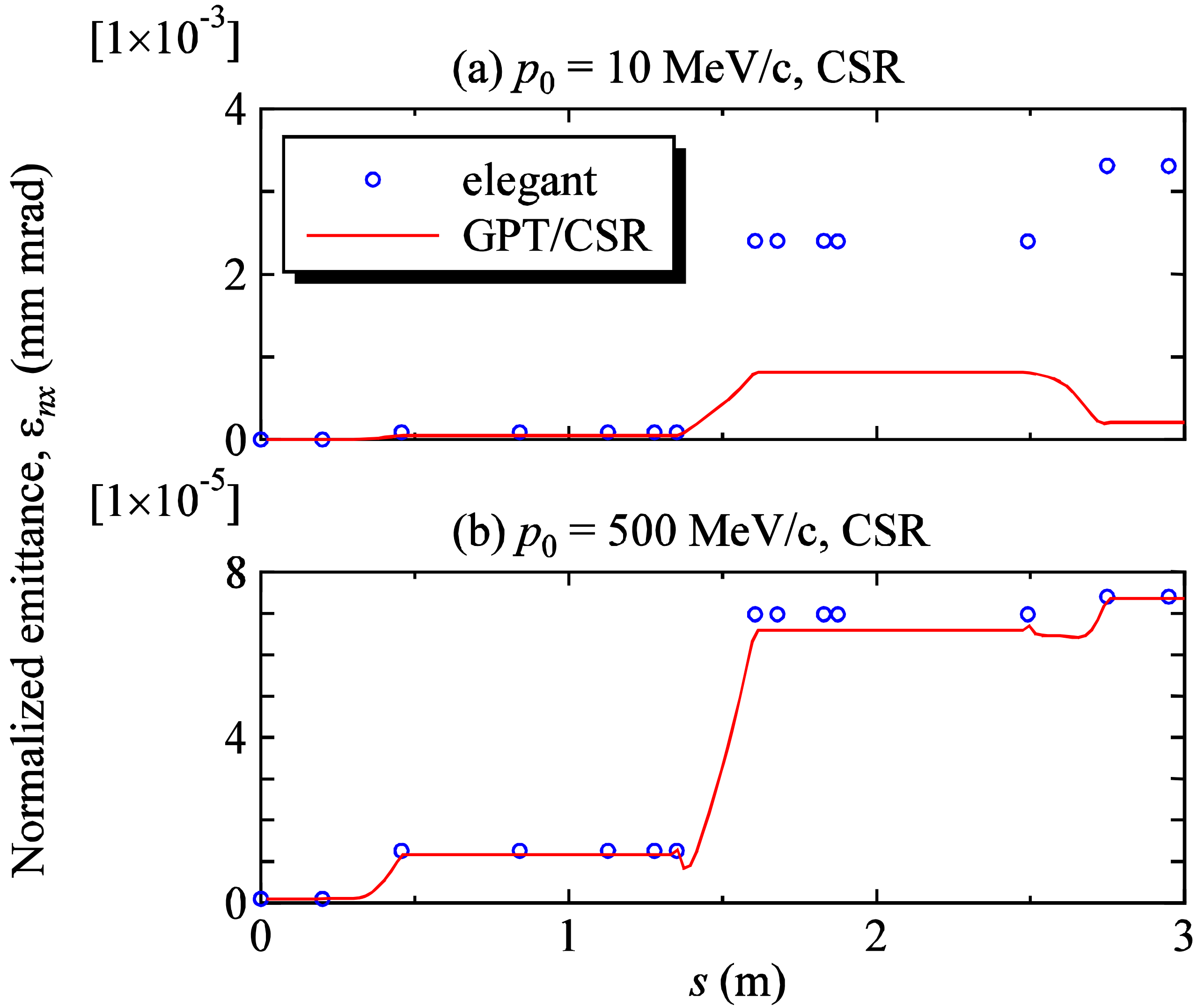
- As an example, the transverse emittance in a 3-dipole merger of ERL project at Cornell University is calculated by GPT/CSR and elegant for two different conditions:
- (a) $p_0 = 10$ MeV/c and (b) $p_0 = 500$ MeV/c.



- Bunch length: $\sigma_s = 0.3$ mm
- Initial distribution: Gaussian
- Bunch charge: $Q = 80$ pC
- Initial emittance : $\varepsilon_{nx} = 1 \times 10^{-12}$ m rad
- Initial betatron function : $\beta_x = \beta_y = 9$ m
- Without shielding and space charge



[1×10^-3]
(a) p0 = 10 MeV/c, CSR
4
2
0
elegant
GPT/CSR
[1×10^-5]
(b) p0 = 500 MeV/c, CSR
8
4
0
0
1
2
3
s (m)
Normalized emittance, εnx (mm mrad)



- For (a) $p_0 = 10$ MeV/c, the GPT/CSR and elegant results disagree.
- For (b) $p_0 = 500$ MeV/c, the agreement is good demonstrating that GPT/CSR reproduces elegant CSR calculations at higher beam energies as expected.

Enhanced 3D Space Charge Routine in GPT

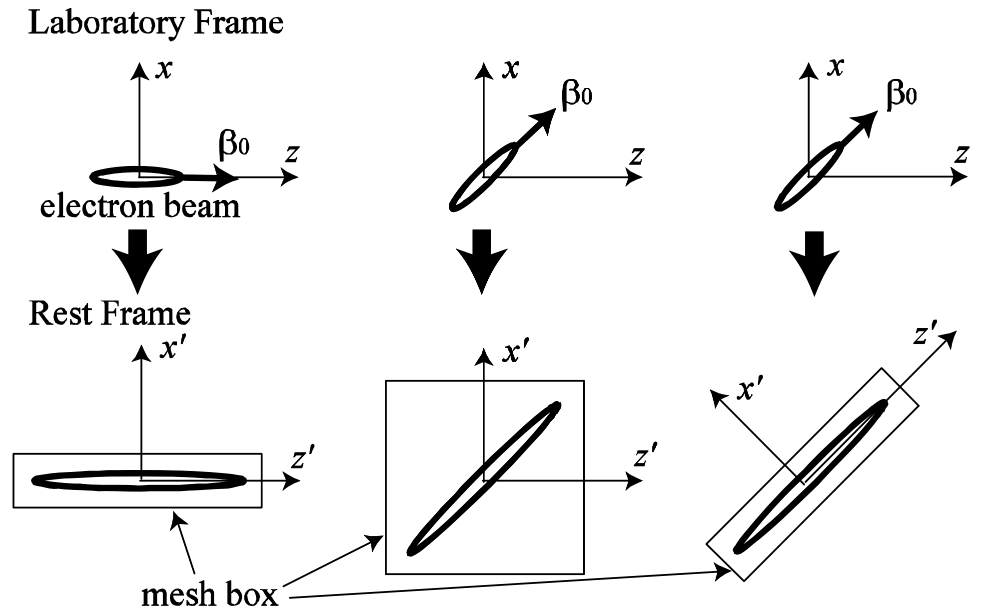
Enhanced 3D Space Charge Routine in GPT

- To calculate the space charge field in the 3D mesh-based routine in GPT, the particle coordinates are transformed from the laboratory frame to the rest frame according to

$$\mathbf{r}'_{\perp} = \mathbf{r}_{\perp}, \quad \mathbf{r}'_{\parallel} = \gamma \mathbf{r}_{\parallel}$$

relative to the direction of motion.

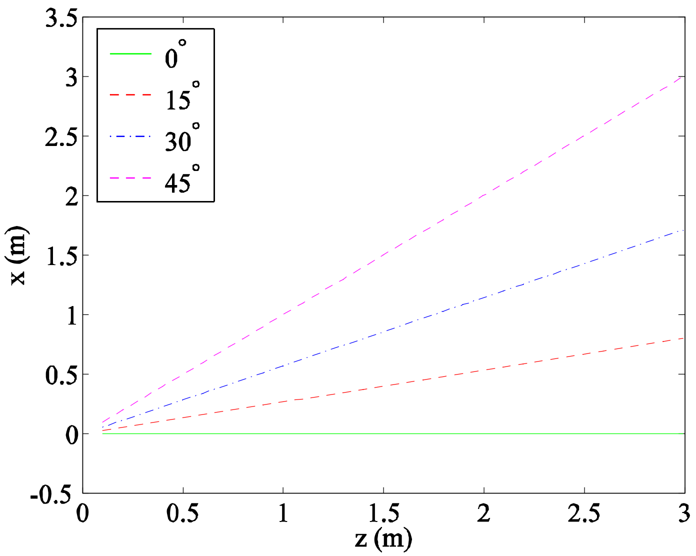
- When the bunch does not move along the z-axis, the bounding box ends up improperly oriented.

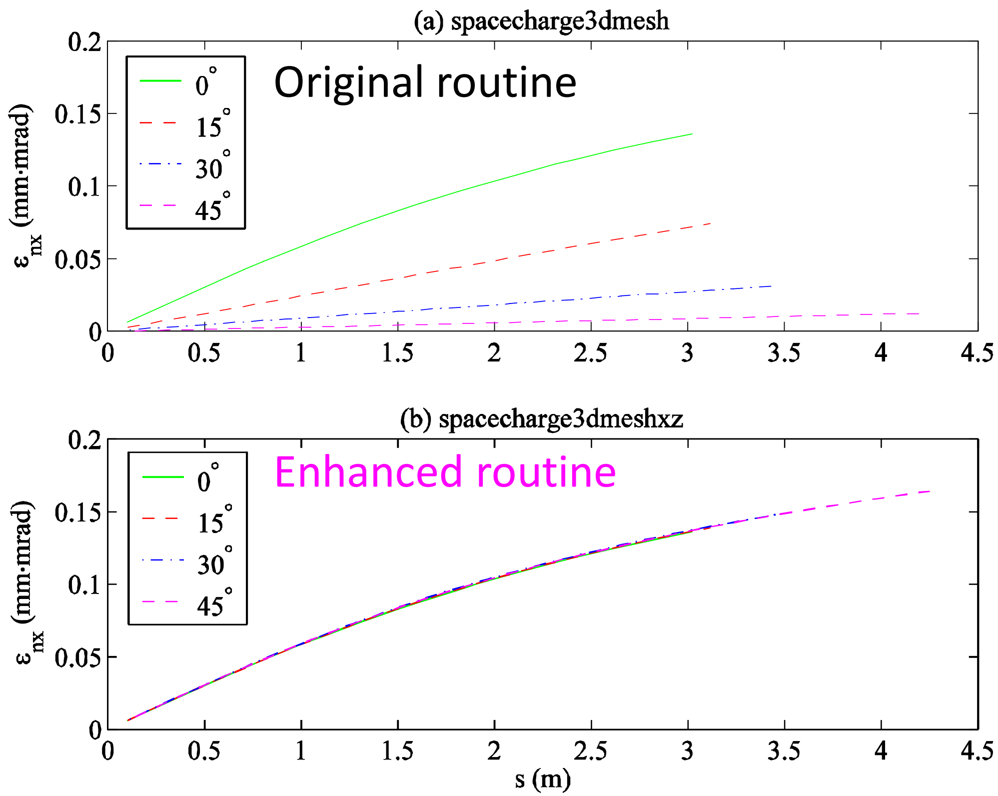


(a) Original routine along z-axis.
(b) Original routine 45 degree direction from z-axis.
(c) New routine 45 degree direction from z-axis.

In this case, for example, the transverse emittance incorrectly depends on the angle relative to the z-axis in a straight trajectory.

To fix this problem, we have added a transformation of rotation in the rest frame in the space charge routine.





Summary

- We have developed a new CSR routine for GPT in order to investigate beam dynamics in ERL and FEL injectors.
- To check GPT/CSR, energy loss and energy spread are calculated by GPT/CSR, elegant and analytical expression.
- The results show GPT/CSR to be effective in a wide range of beam energies.
- We have corrected 3D spacecharge routine in GPT so that it is made applicable to calculating the space charge effect in bending magnets.