Resistive-Wall Multi-Bunch Beam Breakup

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目的

- ERLにおけるresistive-wall wakeの影響を評価して、 ERLプロトタイプの設計に反映させる。
- 今回は、ERLの特徴である大電流連続ビームが引き起こすlong range wakeによるcoupled-bunch BBUについて検討する。
- 特に、longitudinal wakeに比べてビームダクト径の依存性が格段に大きいtransverse wakeの影響を調べる。

Resistive-wall wake

Wake functions (round pipe, ultra-relativistic case β=1)

- Transverse wake(dipole mode)

$$W_{\perp} = -\frac{cL}{\pi b^3 z^{1/2}}\sqrt{\frac{Z_0}{\pi\sigma_c}} \qquad \left(\sqrt[3]{\frac{b^2}{\sigma_c Z_0}} << z << b^2\sigma_c Z_0 \right)$$

- Longitudinal wake(monopole mode)

$$W_{//} = -\frac{cL}{4\pi b z^{3/2}}\sqrt{\frac{Z_0}{\pi\sigma_c}} \qquad \left(\sqrt[3]{\frac{b^2}{\sigma_c Z_0}} << z << b^2\sigma_c Z_0 \right)$$

L: pipe length b: pipe radius σ_c: electric conductivity
Z_0: vacuum impedance (376.73 Ω) z: distance from bunch

smaller b & longer z
→ Transverse wake effect > Longitudinal wake effect

Transverse kick of resistive-wall wake

- Kick angle due to resistive-wall wake per bunch

$$\Delta\theta_y = -\frac{e^2N}{E}W_\perp \cdot y = \frac{e^2N}{E}\cdot\frac{cL}{\pi b^3 z^{1/2}}\sqrt{\frac{Z_0}{\pi\sigma_c}}\cdot y$$

N: Number of electrons per bunch *E*: Beam energy

- Kick angle by the preceding bunch (ERL prototype)

eN=77pC, *E*=200MeV, *z*=0.23m, σ_c(Al)=3.5x10^7 Ω^{-1}m^{-1}

$$\rightarrow \quad \Delta\theta_y[\mathrm{rad}] = 1.42\times10^{-13}\cdot\frac{y[\mu\mathrm{m}]L[\mathrm{m}]}{b[\mathrm{cm}]^3}$$

- Angular divergence of the bunch (ERL prototype)

β_y=10m, ε_y=0.26-2.6 nm·rad ($\varepsilon_{n,y}$=0.1-1.0 mm·mrad)

$$\rightarrow \quad \sigma_{y'}[\mathrm{rad}] = (5-16)\times10^{-6}\ (>>\Delta\theta_y)$$

But number of bunches *M* → ∞ !!

Equation of motion

Equation of motion for an particle in the *M*-th bunch

$$y_M''(s) + k_y^2 y_M(s) = \sum_{N=0}^{M-1} S(M-N) y_N(s)$$

$$S(M) = \frac{a}{\sqrt{M}}, \quad a \equiv \frac{e^2 N}{E} \cdot \frac{c}{\pi b^3 (c\tau_B)^{1/2}} \sqrt{\frac{Z_0}{\pi \sigma_c}} = \frac{4 I_B}{I_A} \frac{\delta_{skin}}{b^3}$$

$$I_B = \frac{eN}{\tau_B}, \quad I_A = \frac{4\pi\varepsilon_0 m c^3 \gamma}{e}, \quad \delta_{skin} = \sqrt{\frac{\tau_B}{\pi \mu_0 \sigma_c}}$$

τ_B: bunch separation in unit of seconds k_y: external focusing
s: position of resistive-wall pipe, s=0 at the entrance

Asymptotic solution for $M \to \infty$ ($M \sim t/\tau_B$)

(J. M. Wang and J. Wu, PRST-AB 7, 034402(2004))

Asymptotic solution (1)

(1) Only the first bunch (M=0) has offset (Initial bunch offset)

$\rightarrow y_0(0) \equiv y_{00} \neq 0$, $y_M(0) \equiv y_{M0} = 0$ $(M \neq 0)$, $y_M'(0) \equiv y_{M0}' = 0$ $(\forall M)$

(A) No focusing case

$$y_M = \frac{y_{00}}{5\sqrt{2\pi}} \left(\frac{t_{NF}}{t} \right)^{9/10} \frac{\tau_B}{t_{NF}} \cdot exp\left[\left(\frac{t}{t_{NF}} \right)^{1/5} \right]$$

$$t_{NF} \equiv \frac{\tau_B}{4\pi} \left(\frac{4}{5} \right)^5 \frac{1}{a^2 L^4} \quad (s = L)$$

(B) Strong focusing case

$$y_M = \frac{2 y_{00}}{3\sqrt{2\pi}} \left(\frac{t_{SF}}{t} \right)^{5/6} \frac{\tau_B}{t_{SF}} \cdot exp\left[\left(\frac{t}{t_{SF}} \right)^{1/3} \right] \cdot cos\left[\sqrt{3} \left(\frac{t_{SF}}{t} \right)^{1/3} - k_y L + \frac{\pi}{6} \right]$$

$$t_{SF} \equiv \frac{16 k_y^2 \tau_B}{\pi} \left(\frac{2}{3} \right)^3 \frac{1}{a^2 L^2} \quad (s = L)$$

Asymptotic solution (2)

(2) All the bunches have the same offset (Injection error)

$\rightarrow y_M(0) \equiv y_{M0} = y_{00} \neq 0 \ (\forall M),\ y_M'(0) \equiv y_{M0}' = 0 \ (\forall M)$

(A) No focusing case

$$y_M = G_{NF} \frac{y_{00}}{5\sqrt{2\pi}} \left(\frac{t_{NF}}{t} \right)^{9/10} \frac{\tau_B}{t_{NF}} \cdot exp\left[\left(\frac{t}{t_{NF}} \right)^{1/5} \right]$$

$$G_{NF} \equiv 5 \left(\frac{t}{\tau_B} \right)^{4/5} \left(\frac{t_{NF}}{\tau_B} \right)^{1/5} = 4 \left(\frac{1}{4\pi a^2} \right)^{1/5} \left(\frac{M}{L} \right)^{4/5}$$

(B) Strong focusing case

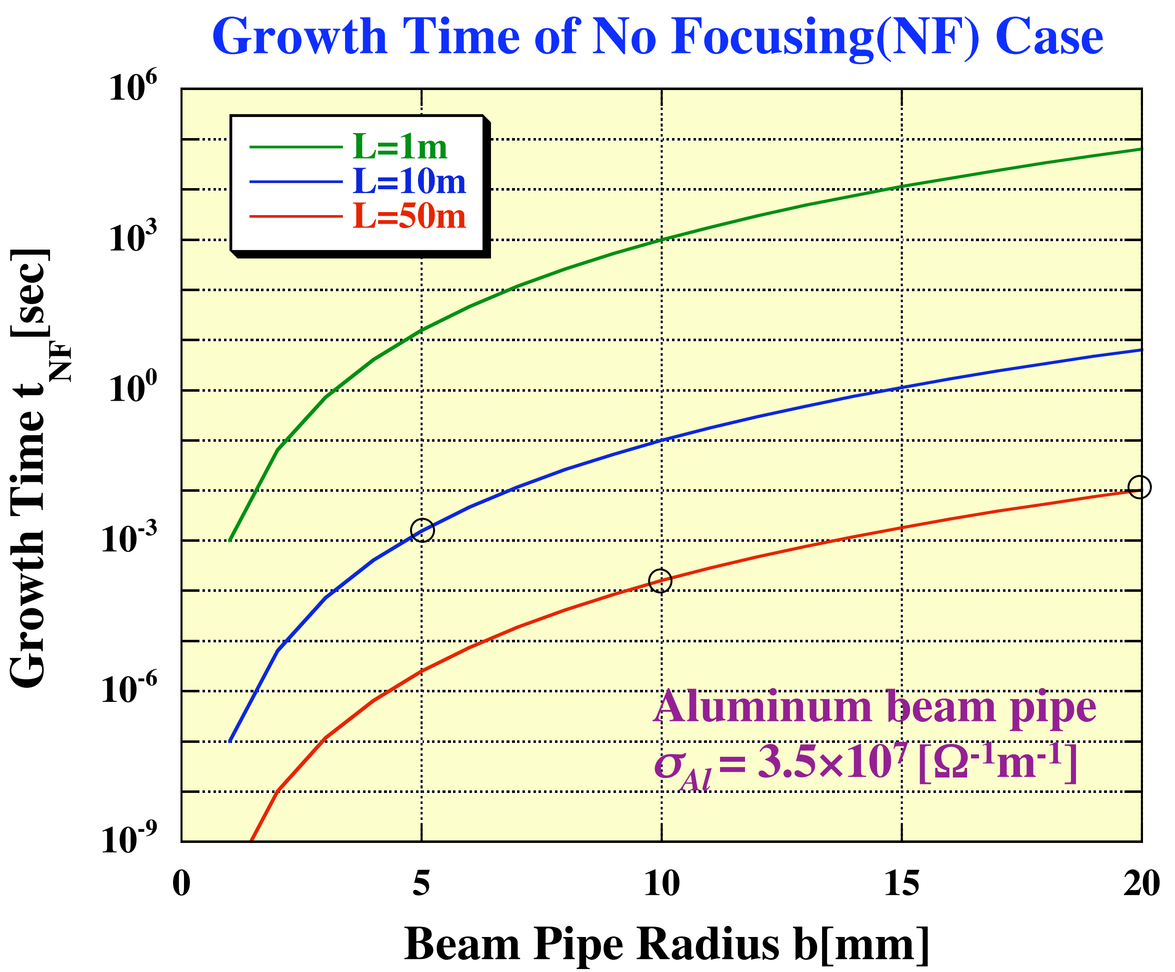
$$y_M = G_{SF} \frac{2y_{00}}{3\sqrt{2\pi}} \left(\frac{t_{SF}}{t} \right)^{5/6} \frac{\tau_B}{t_{SF}} \cdot exp\left[\left(\frac{t}{t_{SF}} \right)^{1/3} \right] \cdot cos\left[\sqrt{3} \left(\frac{t_{SF}}{t} \right)^{1/3} - k_y L - \frac{\pi}{6} \right]$$

$$G_{SF} \equiv \frac{3}{2} \left(\frac{t}{\tau_B} \right)^{2/3} \left(\frac{t_{SF}}{\tau_B} \right)^{1/3} = \left(\frac{16 k_y^2}{\pi a^2} \right)^{1/3} \left(\frac{M}{L} \right)^{2/3}$$

Results of NF case (1)

ERL Prototype: E=200MeV, eN=77pC, $1/\tau_B$=1.3GHz

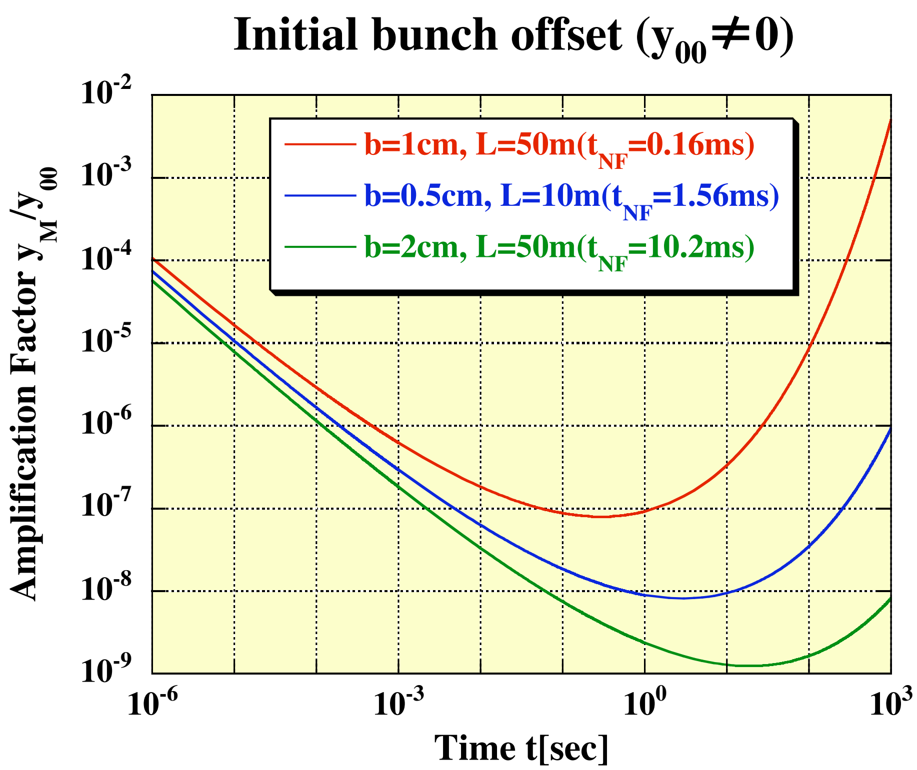
周回部 L~50m　挿入光源部 L~10m

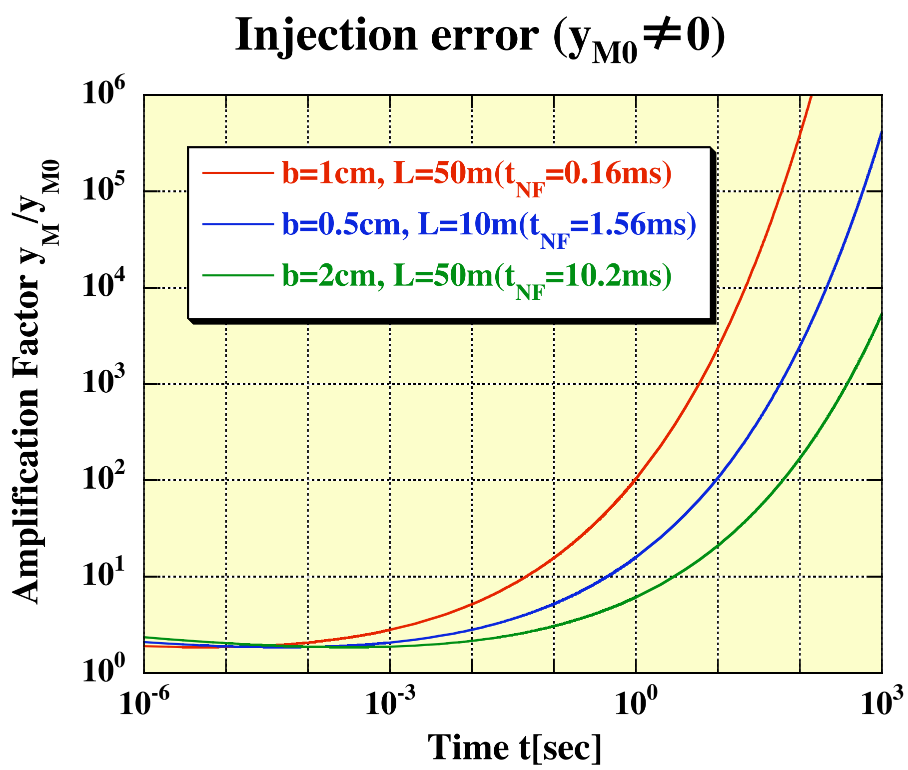


$t_{NF} \propto b^6 L^{-4}$

Results of NF case (2)

Time Dependence of y_M/y_{00} or y_M/y_{M0} (Amplification Factor)



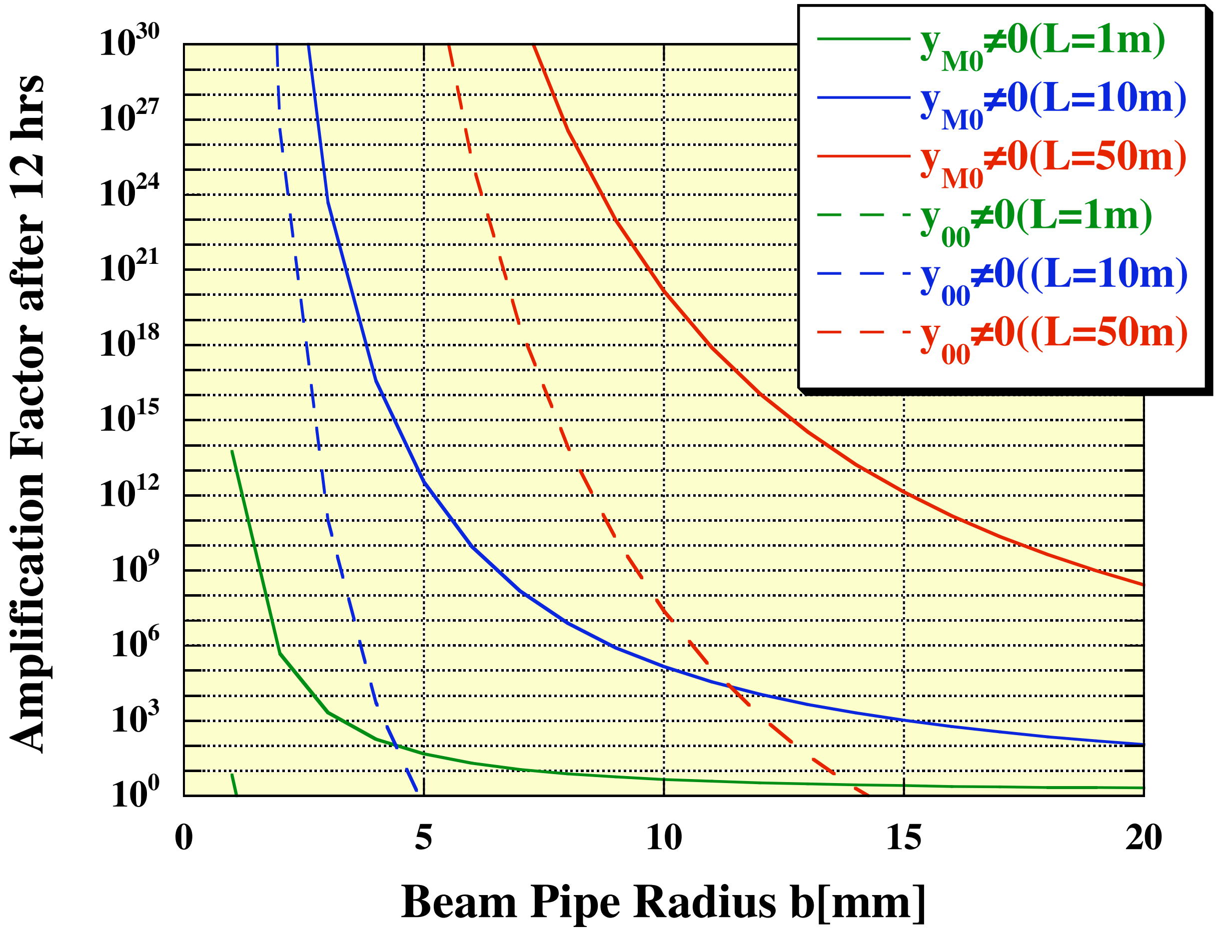


$y_M/y_{00} \propto t^{-9/10} \exp[(t/t_{NF})^{1/5}]$

$y_M/y_{M0} \propto t^{-1/10} \exp[(t/t_{NF})^{1/5}]$

Results of NF case (3)

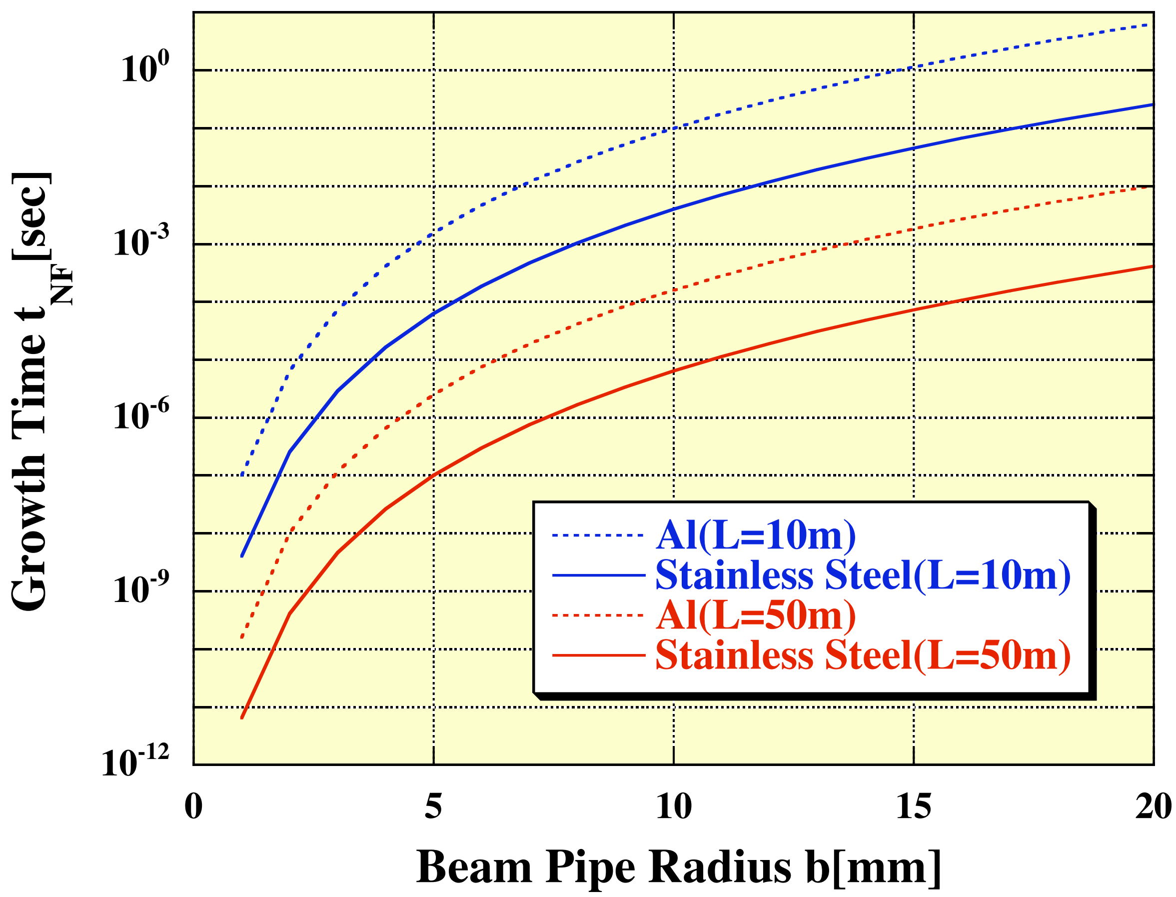
Amplification Factor y_M/y_{M0} (y_M/y_{00}) after 12 hrs



$y_M/y_{00} \propto L^{2/5} b^{-3/5} \exp[c_1 L^{4/5} b^{-6/5}]$, $y_M/y_{M0} \propto L^{-2/5} b^{3/5} \exp[c_1 L^{4/5} b^{-6/5}]$

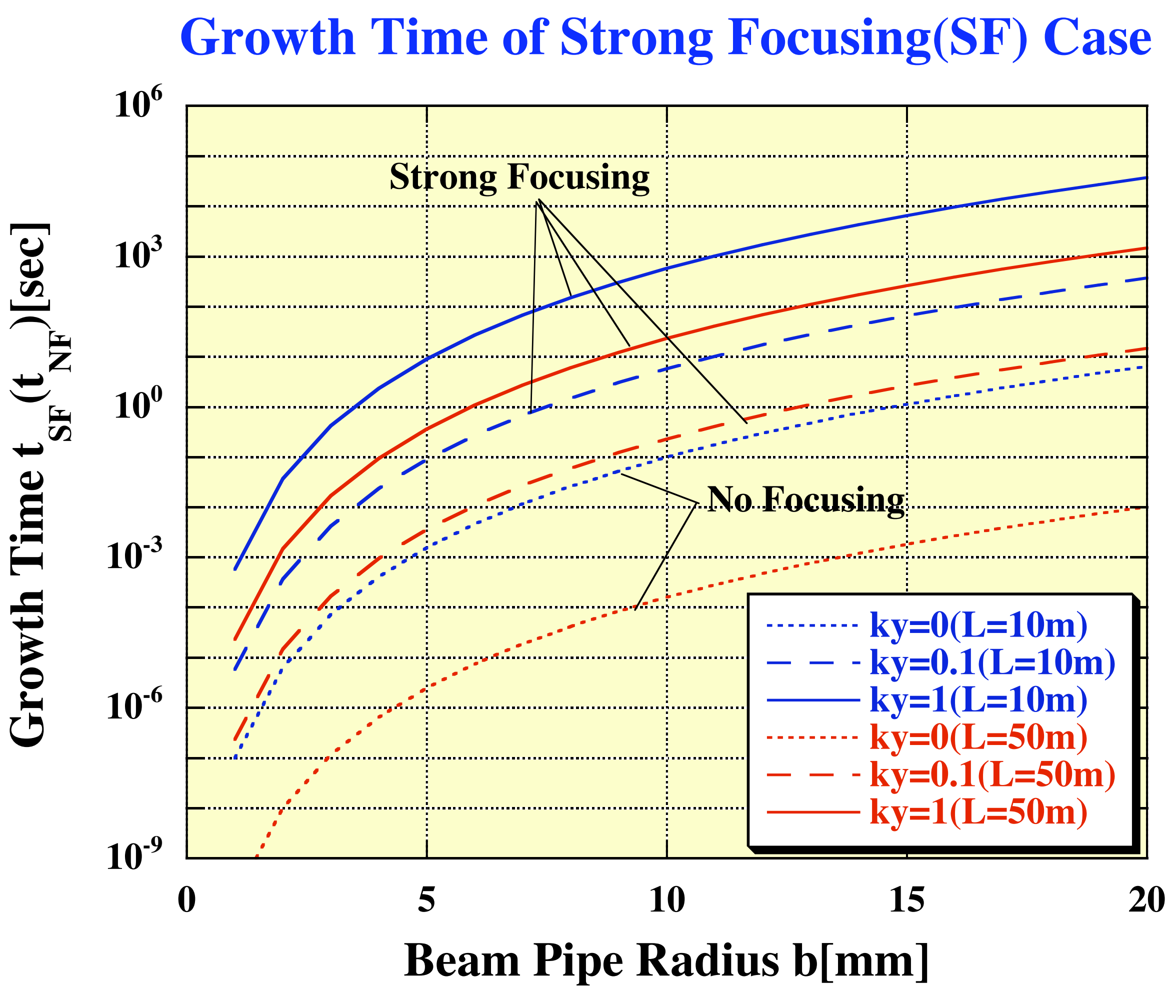
Effect of Electric Conductivity

$\sigma_{SS} = 1.4\times10^{6}\,[\Omega^{-1}m^{-1}],\ \ \sigma_{Al} = 3.5\times10^{7}\,[\Omega^{-1}m^{-1}]$



$t_{NF} \propto \sigma_c$

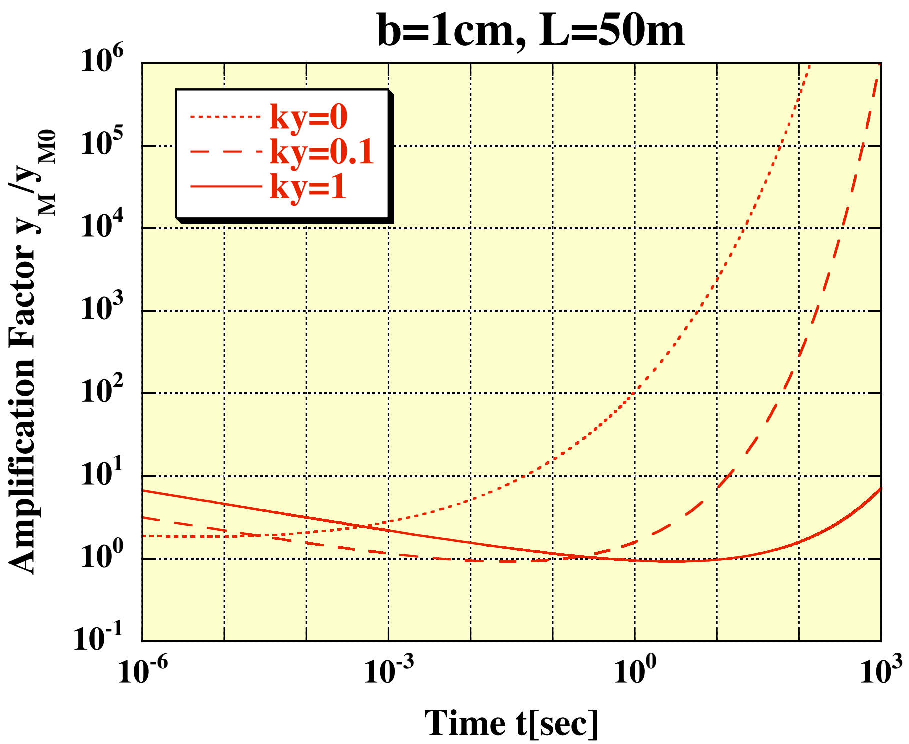
Results of SF case (1)

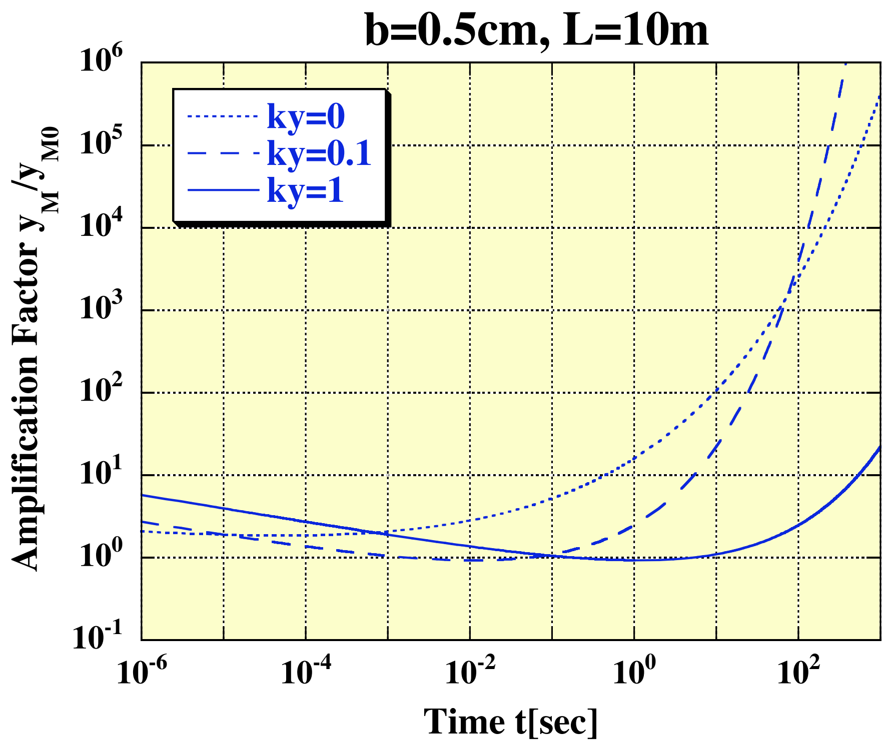


$t_{SF} \propto k_y^2 b^6 L^{-2}$ ($t_{NF} \propto b^6 L^{-4}$)

Results of SF case (2)

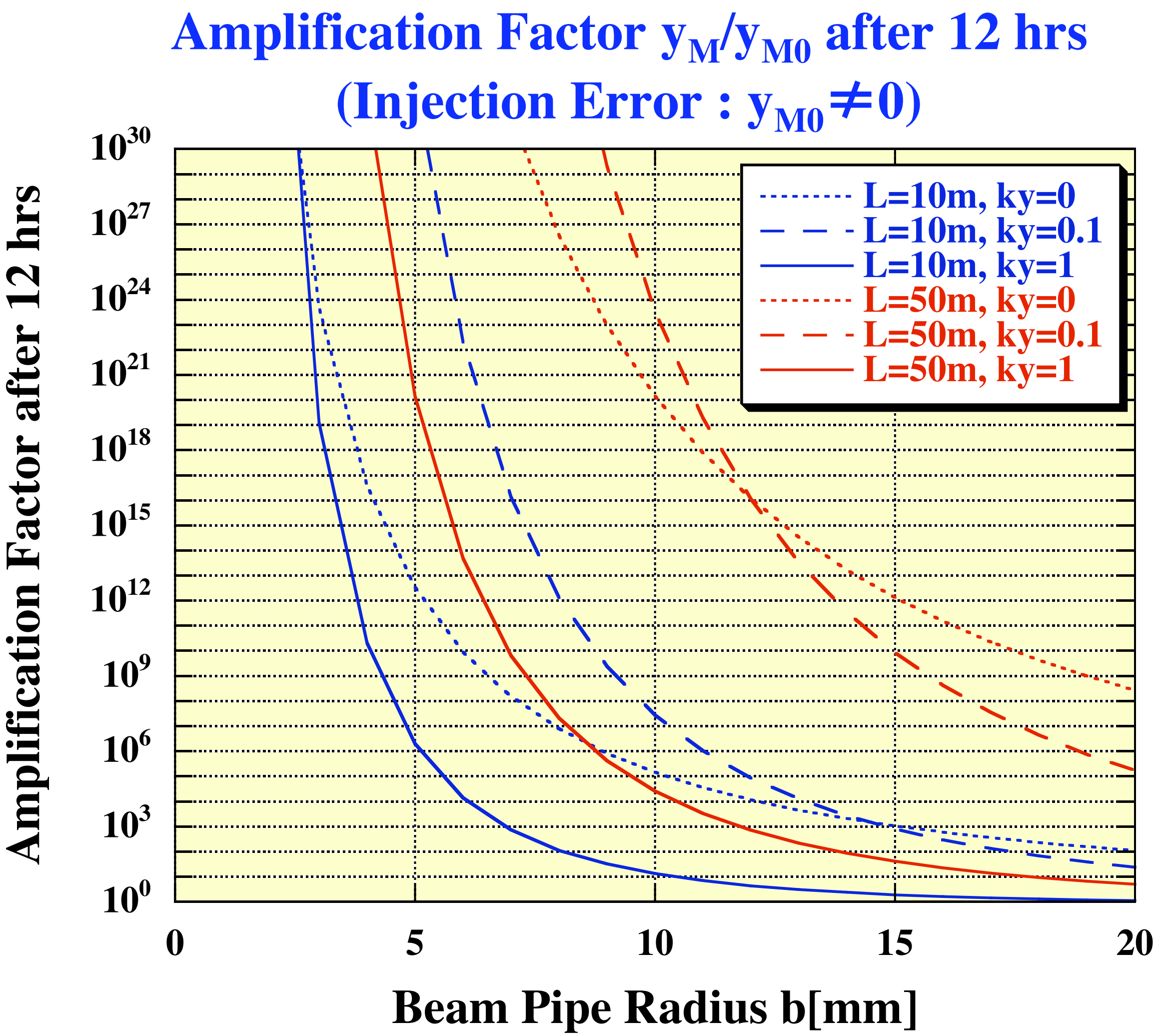
Time Dependence of y_M / y_{M0}
(Injection Error : $y_{M0} \neq 0$)





$$y_M/y_{M0}(SF) \propto t^{-1/6} \exp[(t/t_{NF})^{1/3}]$$

Results of SF case (3)



$$y_M/y_{M0}\,(\mathrm{SF}) \propto L^{-1/3} b k_y^{1/3} \exp[c_2 L^{2/3} b^{-2} k_y^{-2/3}]$$

Parameters for ERL Projects

	KERL	KERL(ID)	BNL-ERL(ID)
Energy [GeV]	5	5	3
RF frequency[GHz]	1.3	1.3	1.3
Bunch charge[pC]	77	77	150
Pipe Length[m]	1253	380	144
Pipe radius[cm]	1.18	0.3	0.3
Conductivity[$\Omega^{-1}m^{-1}$]	3.5×10^{7}	5.9×10^{7}	5.9×10^{7}
Injection time[hrs]	12	12	12
No Focusing Case(k_y=0 m^{-1})			
Growth time t_{NF}[sec]	6.8×10^{-7}	3.7×10^{-8}	1.7×10^{-7}
$y_M/y_{M0}(y_M/y_{00})$	$4\times10^{61}(2\times10^{49})$	$2\times10^{111}(2\times10^{99})$	$6\times10^{81}(4\times10^{69})$
Strong Focusing Case(k_y=3 m^{-1})			
Growth time t_{SF}[sec]	560	2.77	1.83
$y_M/y_{M0}(y_M/y_{00})$	$14(7\times10^{-13})$	$6\times10^{9}(0.0017)$	$2\times10^{11}(0.072)$

まとめ（課題を含む）

(1) Transverse resistive-wall wakeの下では、ビームは何らかのダンピング機構やシステムなしでは生き残れない。ダンピングをどういう方法で実現できるか大きな課題である（例えば、高速フィードバックシステム等）。収束力はBBUを緩和させる効果を持つ。

(2) Transverse wakeの影響はビームのダクト径に大きく依存しており、ERLプロトタイプは実証試験をより円滑に行うためにも安全を考えてなるべく大きなダクト径で設計すべきである。また、ダクト内面の電気伝導度にも注意を払う必要がある。

(3) シュミレーションを行い、解析的な計算結果と比較する必要がある。また、より現実的なあるいは精度の高いシュミレーションを検討する。

(4) ビームを利用したresistive-wall multi-bunch BBUの試験を既存施設で行うことも検討に値するかもしれない（可能であれば）。