Beam dynamics on ERL

1)Introduction

2)Comparison between Krafft's formula and simulation results

a)Transverse Wake Effect

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3)Summary

Introduction

1)Main linac accelerates and decelerates particles on ERL.

Two kinds of beam passes the same linac.

2)We have to consider the lattice that two kinds of beam can pass the linac.

3)There is a threshold current on ERL due to wake effect.

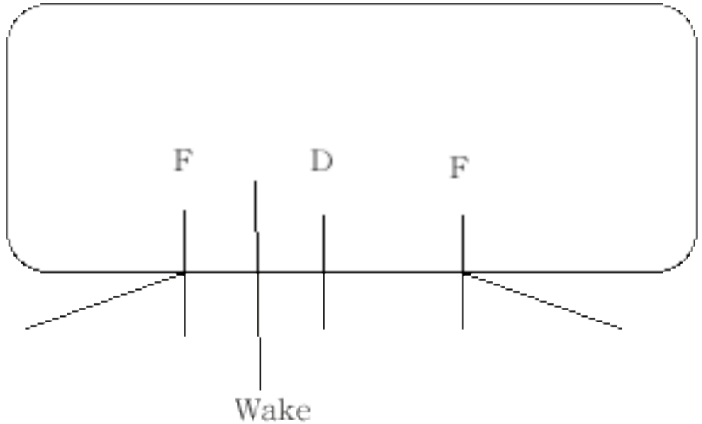
The same particle is the source of wake on the same place of the linac twice.

4)We consider the lattice to reduce the wake effect
to increase the threshold current.

Comparison between Krafft's formula and simulation results

Krafft formula for recirculating linac

System：
One Wake at One Position
No Acceleration



Wake is given by: $W(t) = W_0 e^{-\frac{\omega}{2Q}t} \sin\omega t, \omega = 2\pi f$

1)Beam is always stable for $R_{12} \sin\omega T > 0$

R_{12}:(1,2)componet of transfer matrix from wake to wake.

T:circulation time

2)There is a threshold current $I_{th} < \dfrac{E\square}{W_0 Q\,|R_{12}\sin\omega T\,|}\ for R_{12}\sin\omega T < 0$

E;beam energy

I=2.5mA

E=500[MeV]

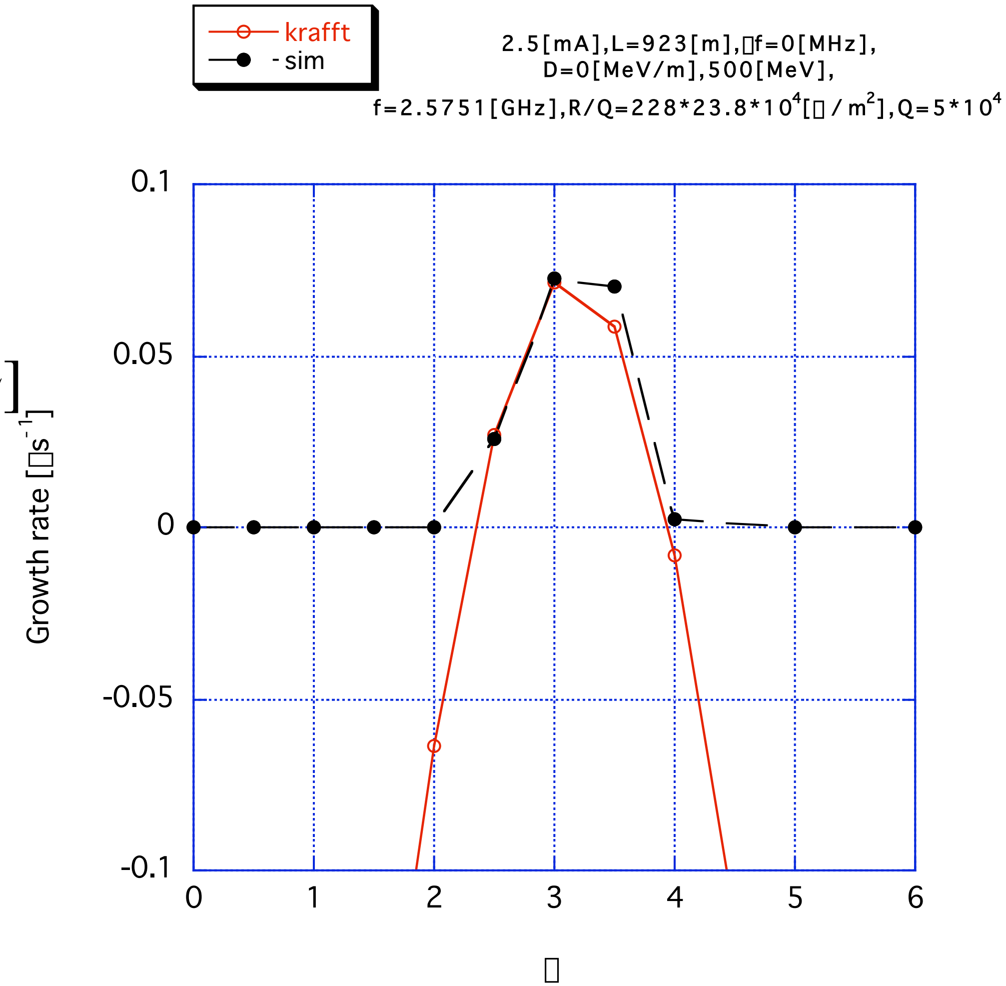
No accel.

R/Q=228*23.8*10^4[□/m^2]

f=2.5751[GHz]

Q=5*10^4

Ψ :phase advance of beta function from wake to wake.



<u>Krafft's can reproduce the simulation results for weak current</u>

I=100mA

E=500[MeV]

No accel.

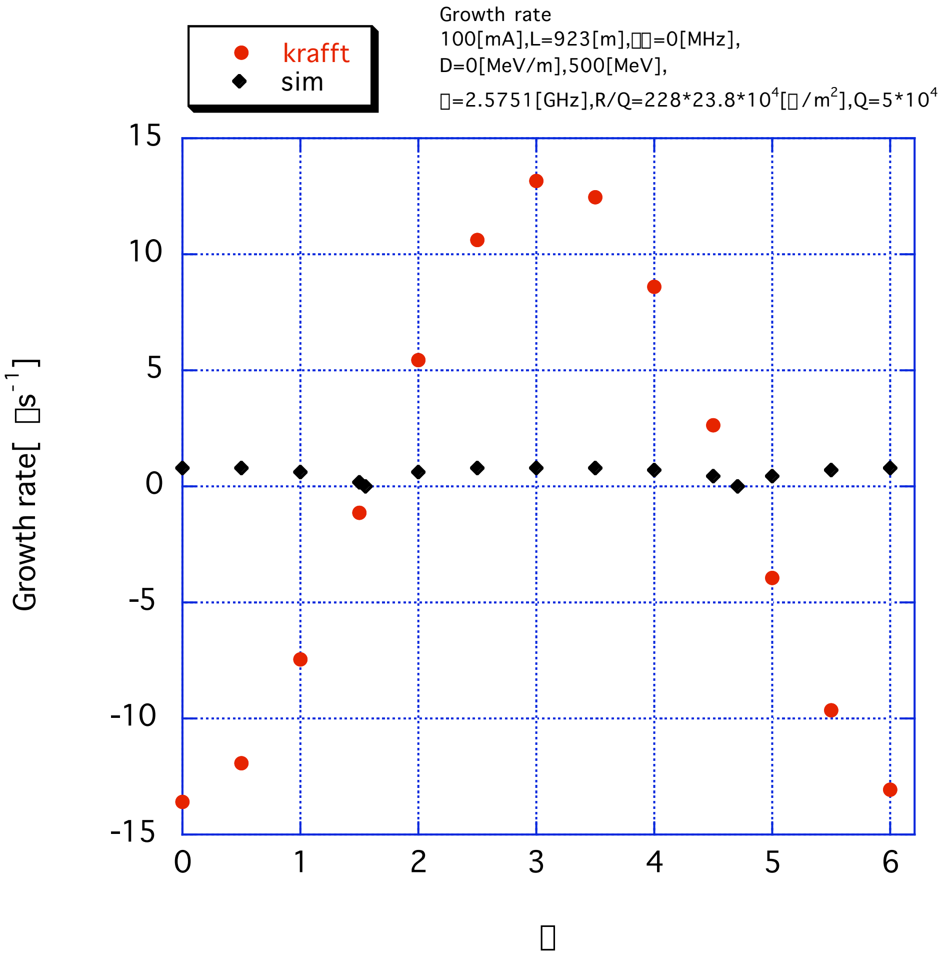
R/Q=228*23.8*10^4[□/m^2]

f=2.5751[GHz]

Q=5*10^4

Ψ:phase advance of beta function

from wake to wake.



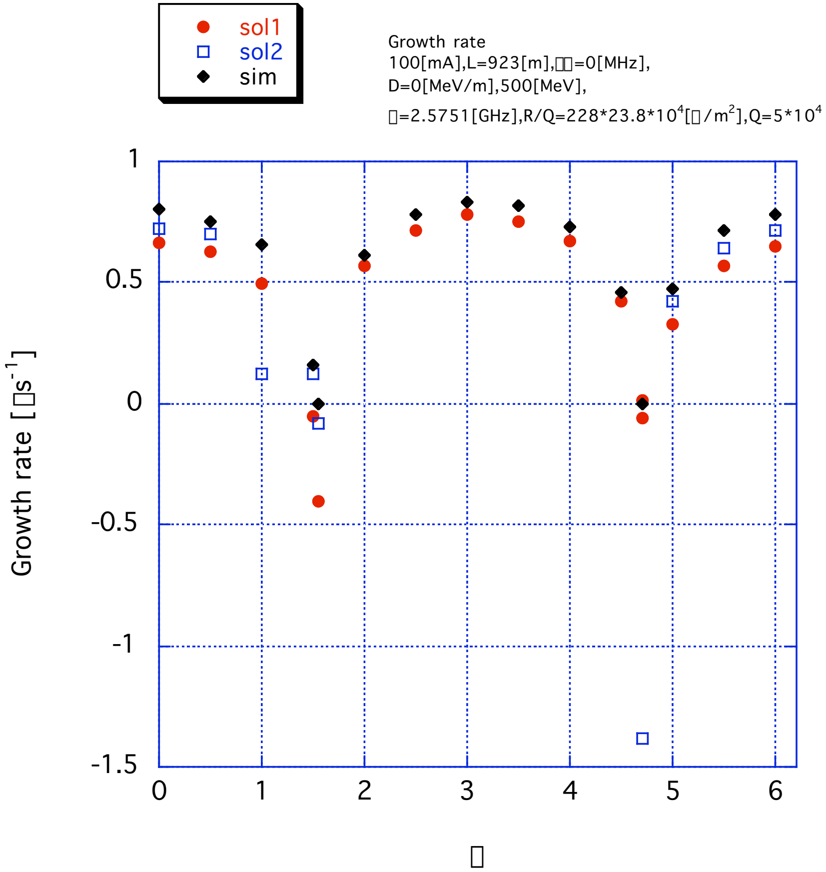
The region of □ where the beam is stable is strictly restricted.

When we use

$$1=\sum_{\pm}\frac{\pm C}{2i}e^{i(m/\square+1)\square}\frac{\exp\left[\square\left(\frac{\square t_b}{2Q}\right)\left(1\square\frac{1}{2\square}\right)\pm i\square t_b\left(1\square\frac{1}{2\square}\right)\right]}{1\square\exp\left[\square\frac{\square t_b}{2Q}\pm i\square t_b\left(1\square\frac{1}{2\square}\right)+i\square\right]}$$

$$C=\frac{It_bW_0R_{12}}{E}$$

t_b:distance between bunches



<u>We can reproduce the simulation results</u>

There are many cavities on main linac.
This means there are many sources of wake along the linac.
Choosing the appropriate ∏(phase advance of beta function for each cavity(wake)) is very important.

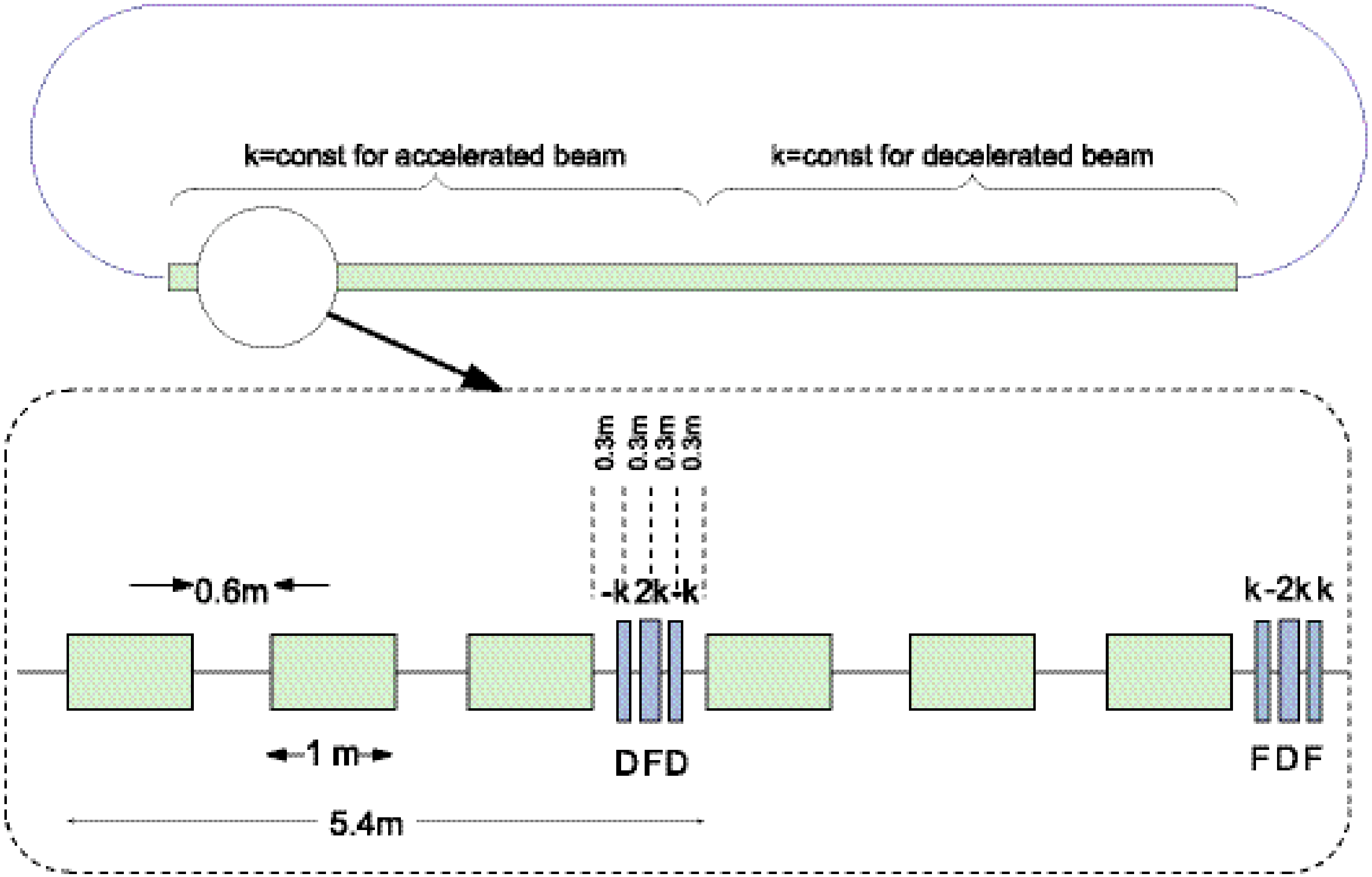
We consider two types of ERL.

1)Magnet focusing linac:

Focusing magnets are configured in order that the strength of focusing force is constant for the lower energy one of the two beams,
i.e. for the accelerating beam in the first half of linac
and for the decelerating beam in the latter half of linac.

2)RFQ focusing linac :

One can use RFQ for beam focusing instead of quadruple magnets.In principle, the focusing force(k-value) can be made equal for the two beams by using two different focusing mode.
This scheme can make the optics functions uniform all over the linac



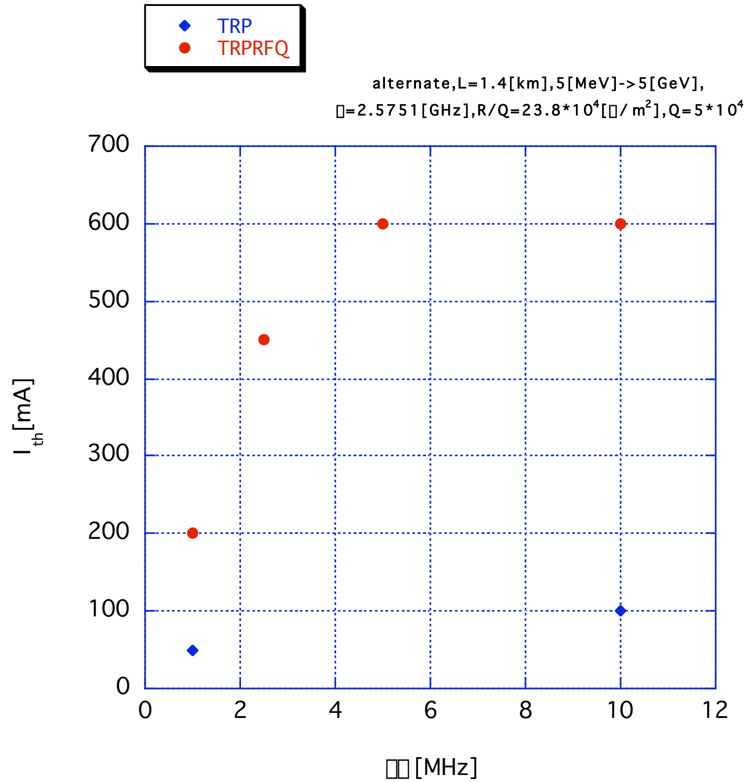
There are 225 cavities along the linac.

a)The Transverse wake Effect

Introducing the spread of the frequency of higher mode wake, we can increase the threshold current.

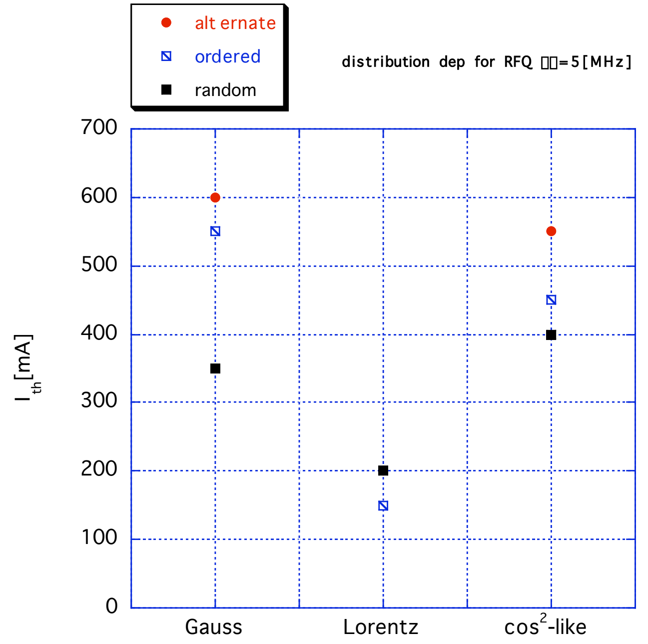
Length of ERL:1.4[km]
R/Q=23.8*10^4[□ /m^2] for one cavity
There are 225cavities along the linac
Frequency f=2.5751[GHz]
Q=5*10^4
Particles are accelerated from 5[MeV] to 5[Gev]

Blue:magnet focusing linac,
Red:RFQ focusing linac

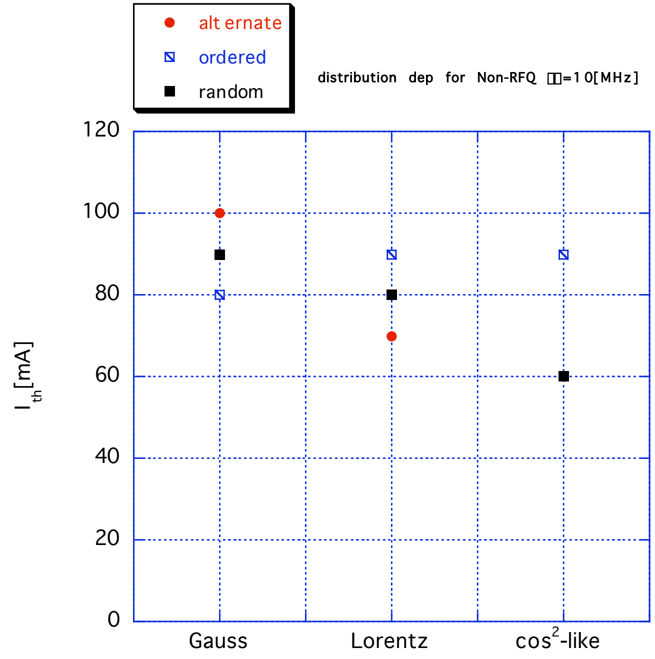


The dependence of the threshold current on the distribution function

Frequency spread of wake obeys a distribution function.
We consider three types of distribution function;
Gaussian, Lorentzian, $\cos^2$-like.



RFQ-focusing linac case



Magnet focusing linac case

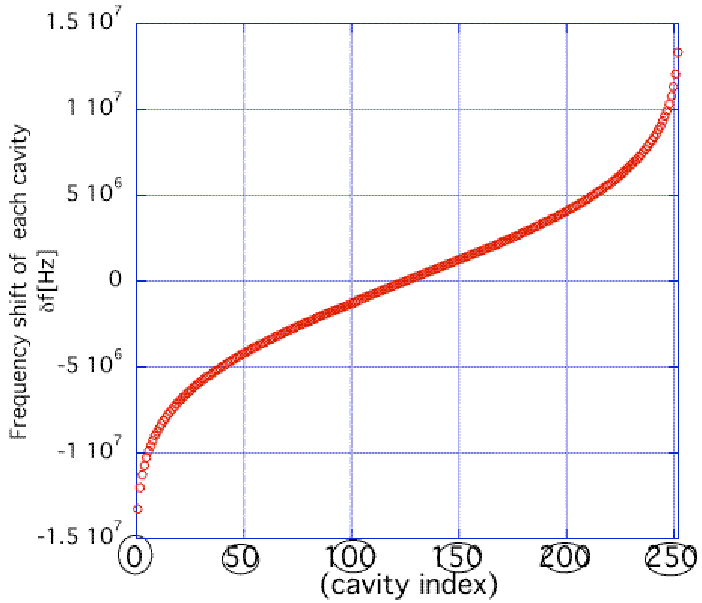
<u>Gaussian seems best.</u>

Alternate,ordered,random represent the ordering of cavities

Configuration of cavities along the linac

The threshold current depends on how the cavities are configured along the linac especially for RFQ focusing linac.

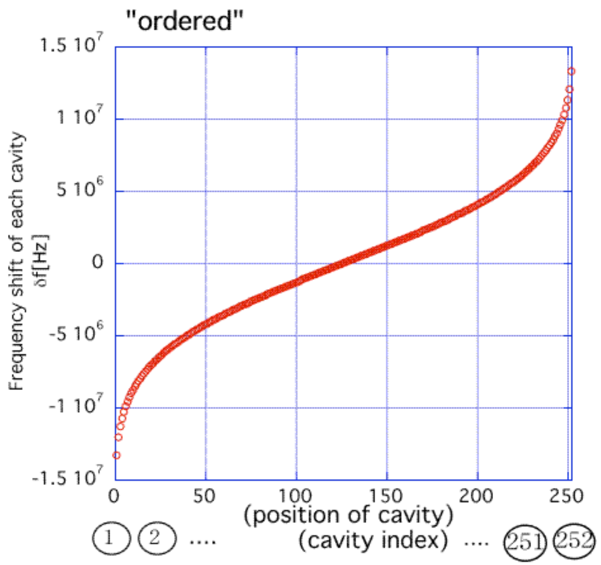
We consider the case that rms of the frequency spread is □f=5MHz.
We create the frequency spread intentionally.

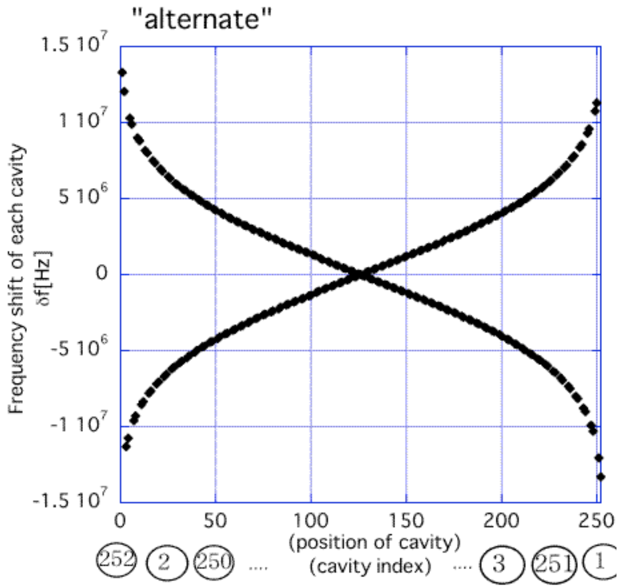


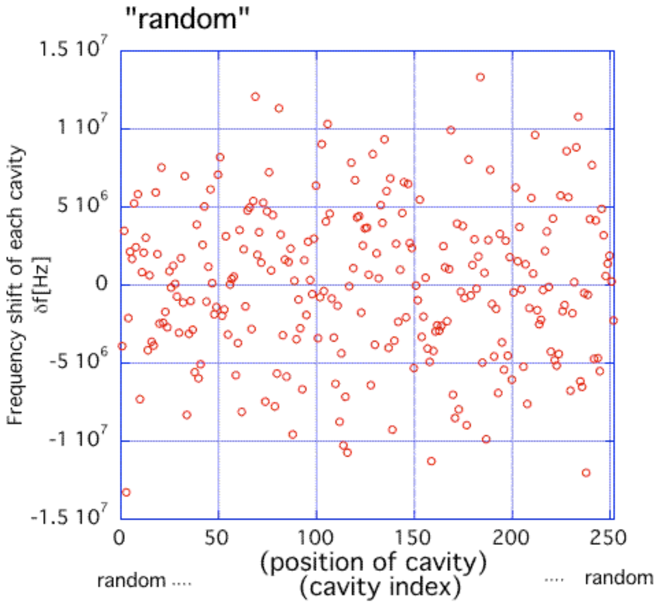
If we assign the cavity index, we can know the frequency of wake.

We consider three types of configuration along the linac.

1)"ordered" : locate the cavities from 1 to 252 from entrance to exit.
2)"alternate" : locate cavities alternately, i.e. cavity#1 at exit end, #2 at second from entrance, #3 at third from exit, etc.
3)"random" : shuffle the cavity indices from entrance to exit by a random number.



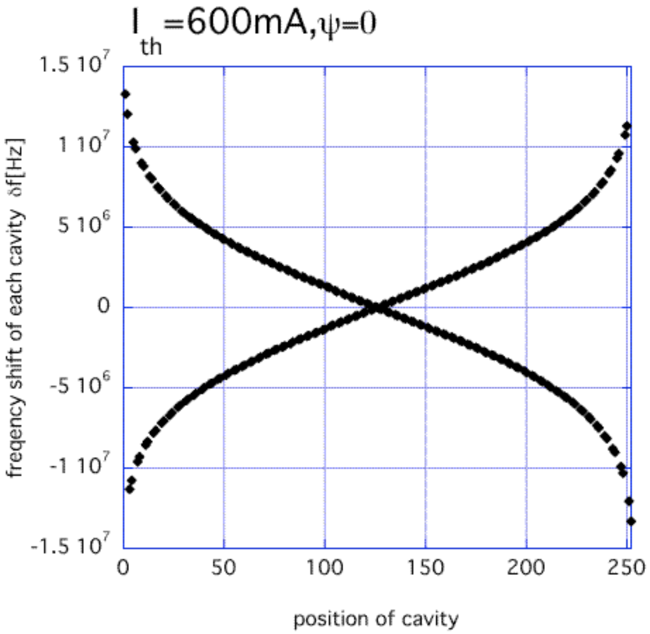


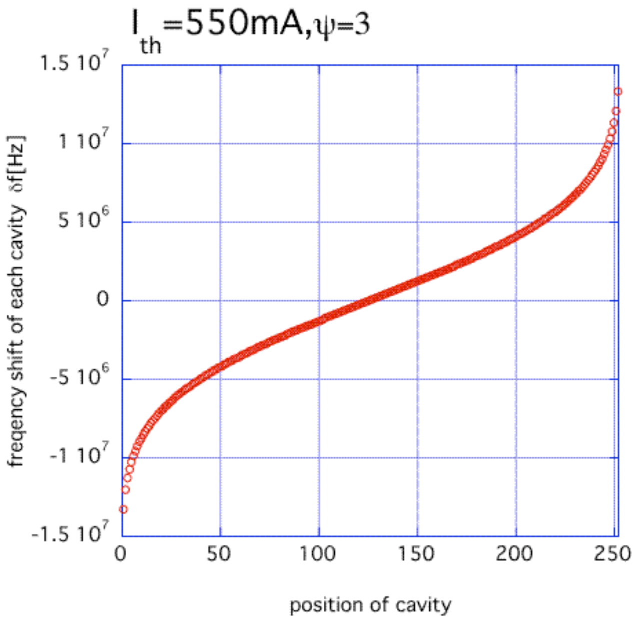


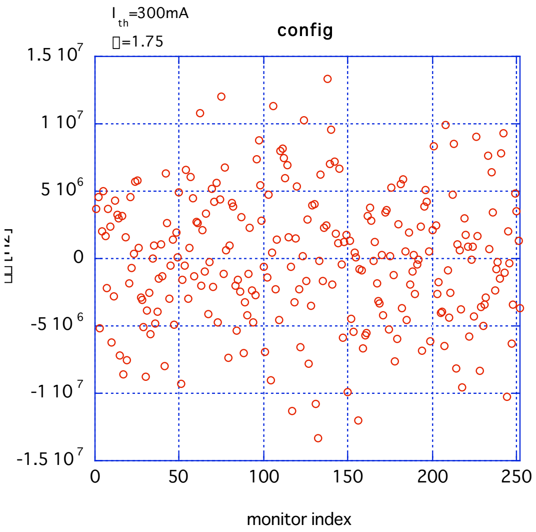
The threshold current is the highest when the configuration is “alternate”

The most appropriate □ is different for each configuration.

350mA,□=1.75







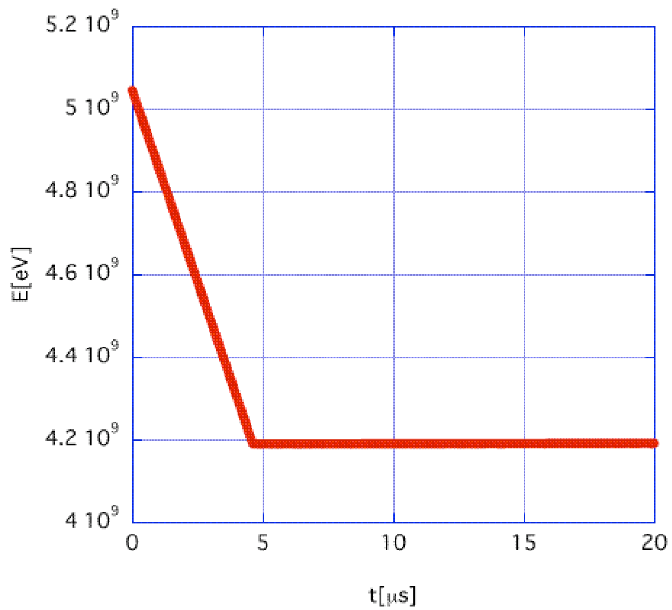
Injection scheme(the effect of the longitudinal wake for the fundamental mode)

We adopt the TESLA type cavity.
R/Q=900[Ω].
Since R/Q is large, transient effect is large.
We observe the beam energy at end of the linac.
The fluctuation of the energy is evaluated as

$$(\text{Number of Cavity}(225)) * I * t_b W \omega T$$

t_b: distance between bunches, W: strength of wake, $\omega = 2\pi f$, T: circulation time

According to our simulation, beam current must be increased by 0.1(Q/f)[mA/s].
Otherwise, beam loss occurs.



Summary

1)We consider two types of ERL.

one is magnet focusing linac and the another is RFQ focusing linac.

2)The threshold current for RFQ focusing linac is higher than that for magnet focusing linac.

3) The threshold current is increased most efficiently when this frequency spread obeys the Gaussian distribution.

We create the spread of frequency of higher order mode wake intentionally.

4) The threshold current depends on how the cavities are configured along the linac especially for RFQ focusing linac.

5) The threshold current is the highest when the cavities are configured “alternately”.

Basically, the frequency deviation must be large at both the entrance and exit of linac.