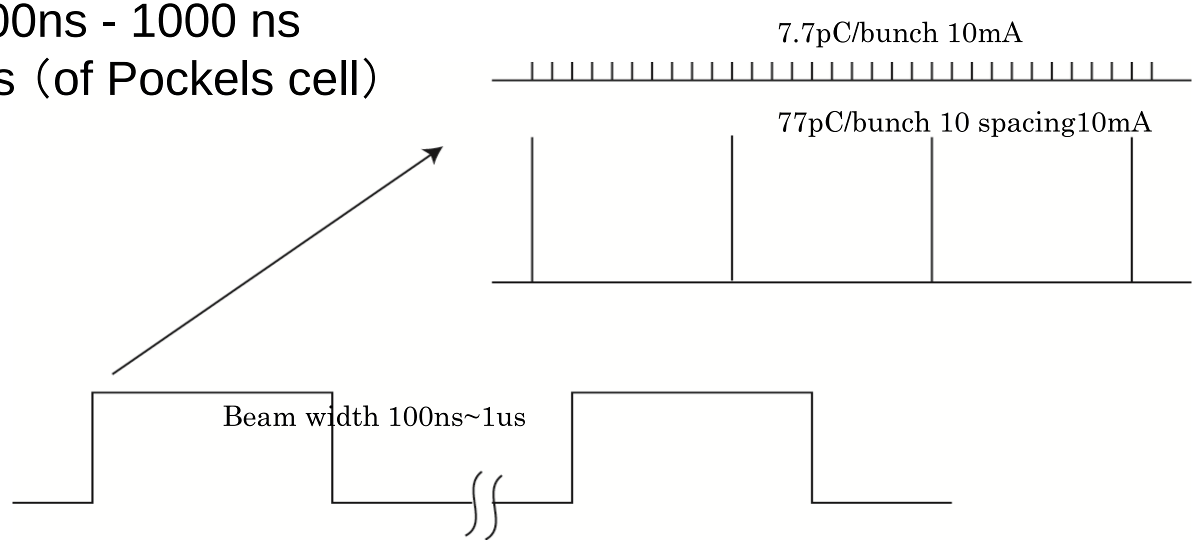
Beam loading during "burst mode" operation

道園真一郎

Burst mode parameters
Cavity equation
Without beam loading compensation at Injector
FF compensation (with jitter) at Injector
Main linac

"burst mode" operation

Burst mode parameters
Bunch charge:7.7pC/bunch（77pC/bunch）
7.7 pC/bunch=10mA at 1.3GHz CW
Pulse width:100ns - 1000 ns
Rise time:10ns（of Pockels cell）



Other cavity parameters used for the evaluation

Item	Unit	Buncher	Inj-1	Inj-2	Inj-3	ML-1	ML-2
Structure		NC	SC	SC	SC	SC	SC
Energy Gain	MV	0.14	1	2	2	15	15
Vcav			1.15	2.03	2.03	15	15
QL			5E+05	2E+05	2E+05	2E+07	2E+07
R/Q	ohm		200	200	200	1000	1000
Ig	mA		23.1	101.5	101.5	1.5	1.5
Power Required	kW	4.5	10	37	37	11	11
Beam Phase	deg.	-90	-15 to -30	-10	-10	0	0

Cavity equation

$$\frac{d}{dt}\begin{pmatrix} V_r \\ V_i \end{pmatrix} = \begin{pmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} R_L\omega_{1/2} & 0 \\ 0 & R_L\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} I_r \\ I_i \end{pmatrix}$$

Here, $\Delta\omega = \omega_0 - \omega$, $\omega_{1/2} = \omega_0/2/Q_{L.}$ (and $R_l I_r = V_g$, $R_l = 1/2R/Q$ Ql etc.)
When we use complex ($V = V_r + jV_i$),
$\acute{V} = -\left(\omega_{\frac{1}{2}} - j\Delta\omega\right)V + \omega_{\frac{1}{2}}(V_{gr} + V_{br})$ (2)
Here, we evaluate the llrf performance during burst mode operation (pulsed beam operation)
We suppose the detuning to be zero (Dw=0)
$\acute{V} = -\omega_{\frac{1}{2}}V + \omega_{\frac{1}{2}}R_l\left(I_g - I_b\right) = -\omega_{\frac{1}{2}}V + \omega_{\frac{1}{2}}\left(\frac{1}{2}\frac{r}{Q}Q_L\right)\left(I_g - I_b\right)$ (3)
During the no beam condition (and steady state)

$$V = R_l I_g = \frac{1}{2}\frac{r}{Q}Q_L I_g$$

$$I_g = {V}\Big/{\frac{1}{2}\frac{r}{Q}Q_L}$$

These generator currents (Ig) are also shown in Table.1.

ベクトル図

- 超伝導空洞の場合は，ビームローディングが大きな割合を占める.
- 下記のベクトル図は，定常状態の場合で，時間変化する系では一部正しくないことに注意が必要.

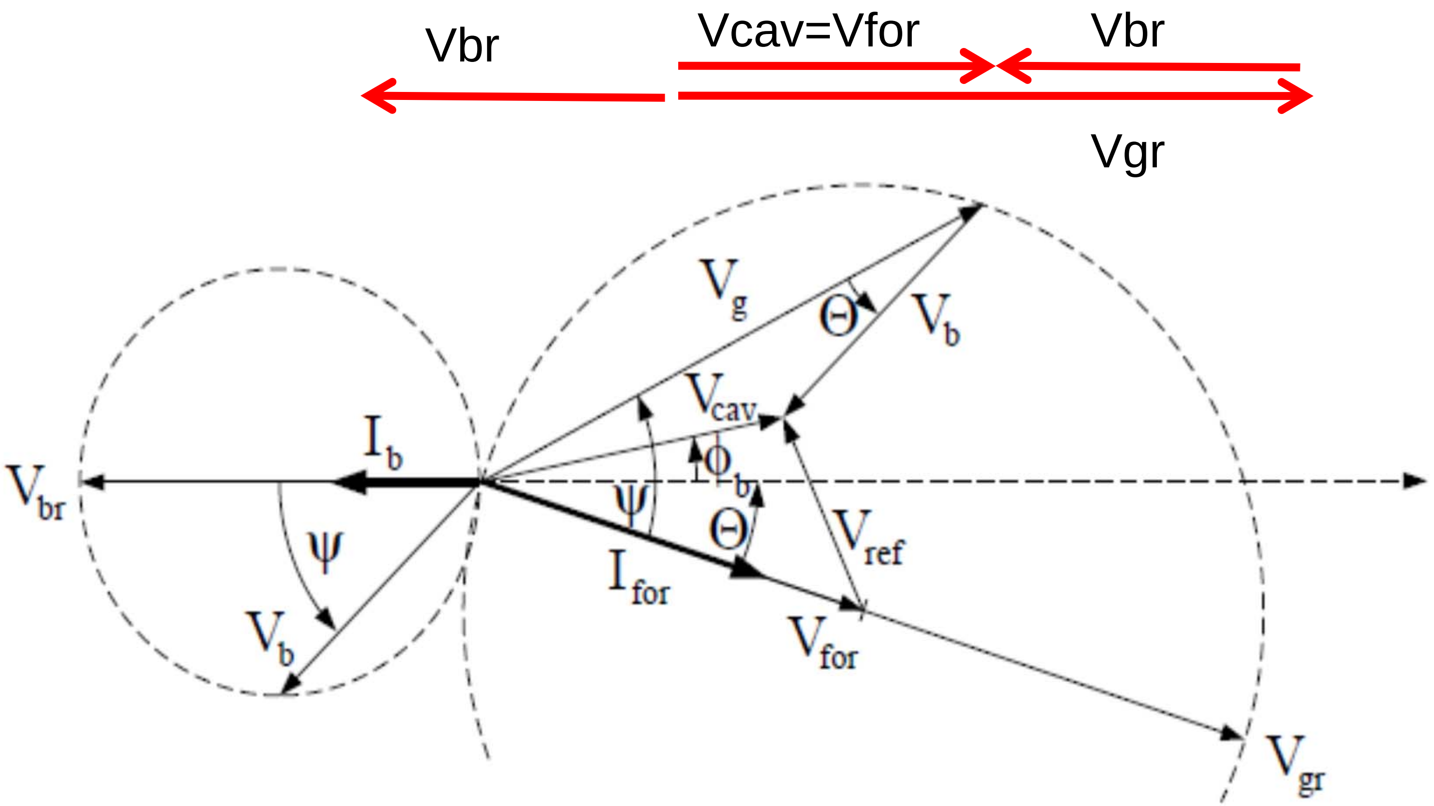


Figure 3.9: Vector diagram of generator- and beam-induced voltages in a detuned cavity. The angle ϕ_b denotes the beam phase and ψ the tuning angle.

Without beam loading compensation (no FF beam)

$V_0 = \left(\frac{1}{2}\frac{r}{Q}Q_L\right)I_g$ (4)

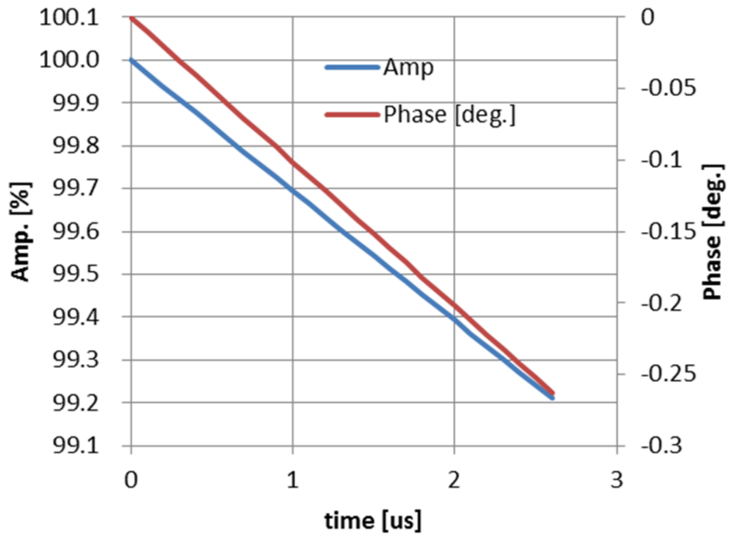
The step response is

$V = \frac{1}{2}\frac{r}{Q}Q_L I\left(1 - e^{-\frac{\omega_1 t}{2}}\right)$ (5)

Thus the cavity voltage decrease;$V = V_0 - \frac{1}{2}\frac{r}{Q}Q_L I_b\left(1 - e^{-\frac{\omega_1 t}{2}}\right)$

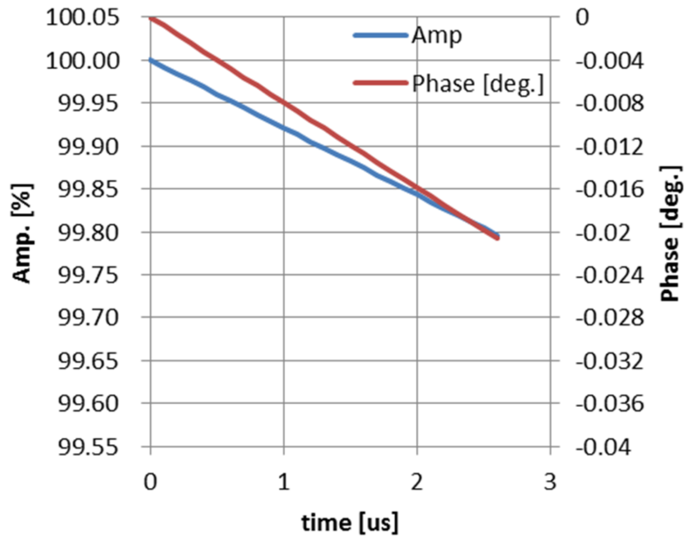
Ib: complex number.

Injector 1(Ig=23.1mA, Ib=10mA, beam phase=30deg.)



Amplitude and phase change after 1us-long beam are 0.30% and 0.10deg., respectively.

Injector 2&3 (Ig=101.5mA, Ib=10mA, beam phase=10deg.)



Amplitude and phase change after 1us-long beam are 0.09% and 0.009deg., respectively.
Rather smaller effects than Injector 1 result from the high Ig and smaller beam phase.

Higher charge (77pC/bunch, 10 bucket spacing) will show same tendency because the 10 buckets are too short for the superconducting cavities.

With beam loading compensation (with different compensation accuracy)

Since the typical digital feedback loop time is ~1us, the beam loading cannot be compensated by feedback.

By adding the perfect feed forward synchronized to the beam, we can compensate rf field perfectly. But because of the limitation of synchronization (such as digital system clock (~40MHz), jitter of the beam timing), there is some residual llrf error.

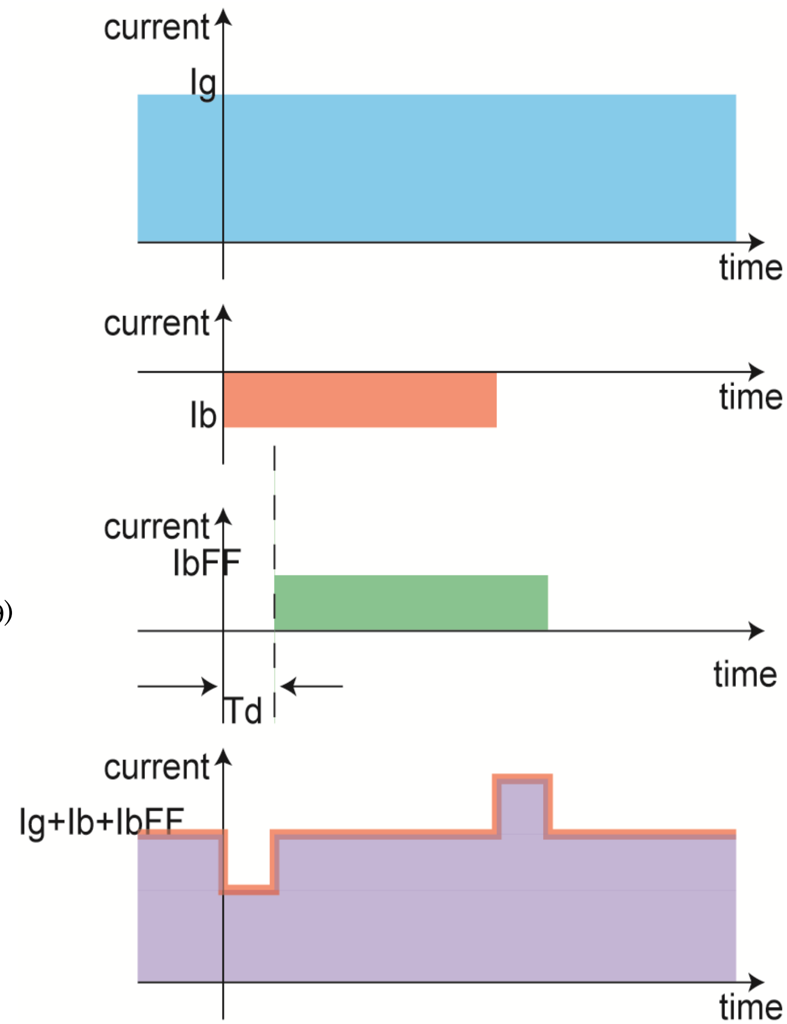
$$\acute{V} = -\omega_{\frac{1}{2}} V + \omega_{\frac{1}{2}} \left(\frac{1}{2}\frac{r}{Q} Q_L\right)\left(I_g - I_b(t) + I_{bFF}(t - T_d)\right) \quad (6)$$

The solution is

$$t<0:\ V = \frac{1}{2}\frac{r}{Q} Q_L I \left(1 - e^{-\omega_{\frac{1}{2}} t}\right) \quad (7)$$

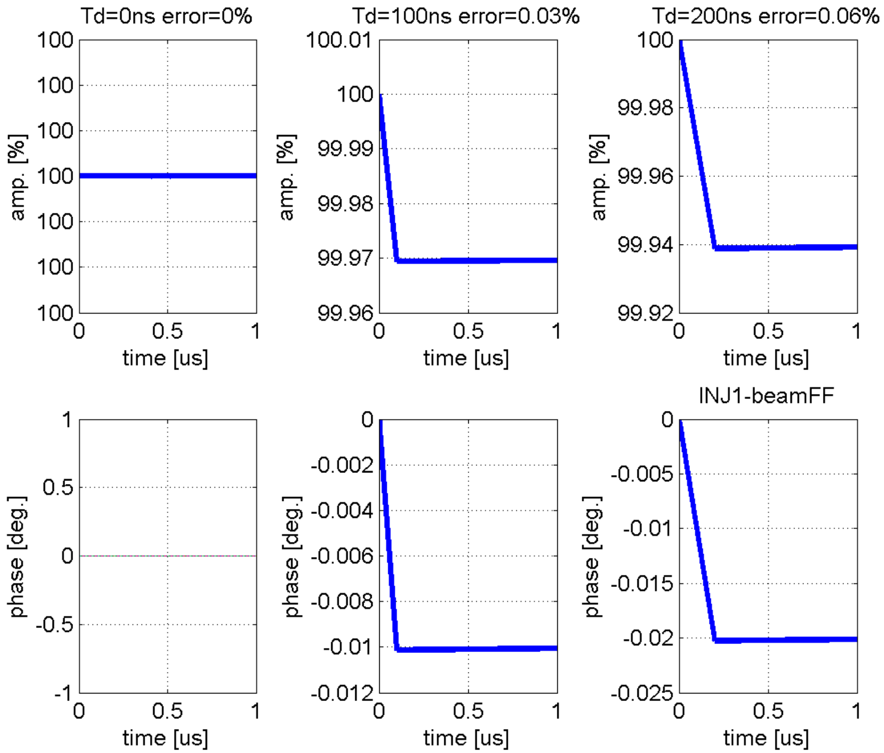
$$t>0:\ V = V_0 - \frac{1}{2}\frac{r}{Q} Q_L I_b \left(1 - e^{-\omega_{\frac{1}{2}} t}\right) \quad (8)$$

$$t>Td:\ V = V_0 - \frac{1}{2}\frac{r}{Q} Q_L I_b \left(1 - e^{-\omega_{\frac{1}{2}} t}\right) + \frac{1}{2}\frac{r}{Q} Q_L I_{bFF} \left(1 - e^{-\omega_{\frac{1}{2}} (t - T_d)}\right) \quad (I_b = I_{bFF}) \ (9)$$



With beam loading compensation (with different compensation accuracy)

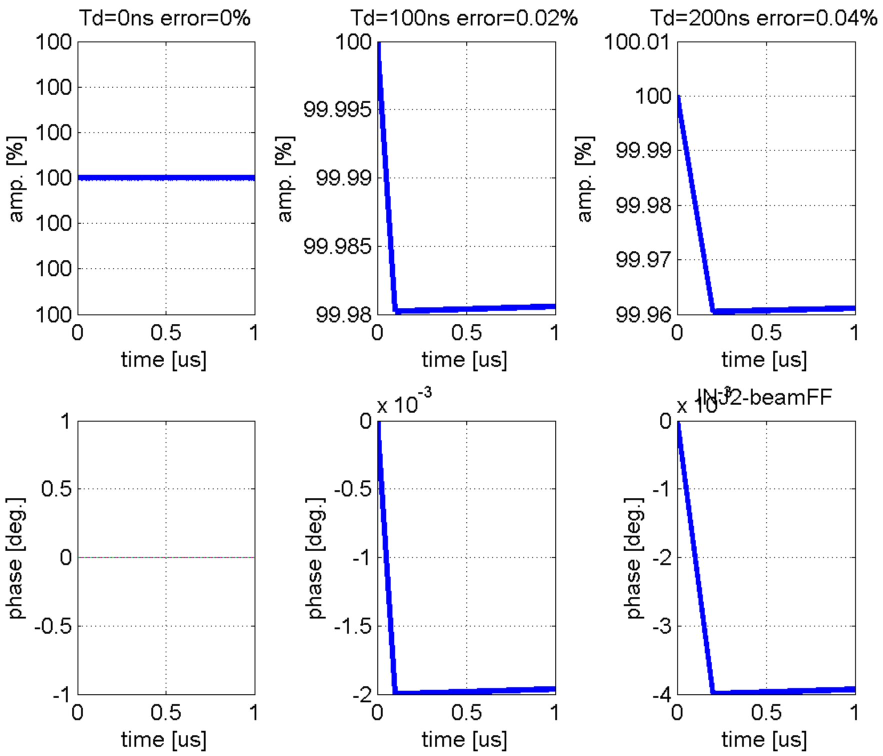
Injector 1(Ig=23.1mA, Ib=10mA, beam phase=30deg., IbFF=10mA (-30deg.))
ERL_burstInj1.m



Even if the jitter (between feedforward and beam) is 200ns, amplitude and phase errors are 0.06% and 0.02deg.

With beam loading compensation
(with different compensation accuracy)

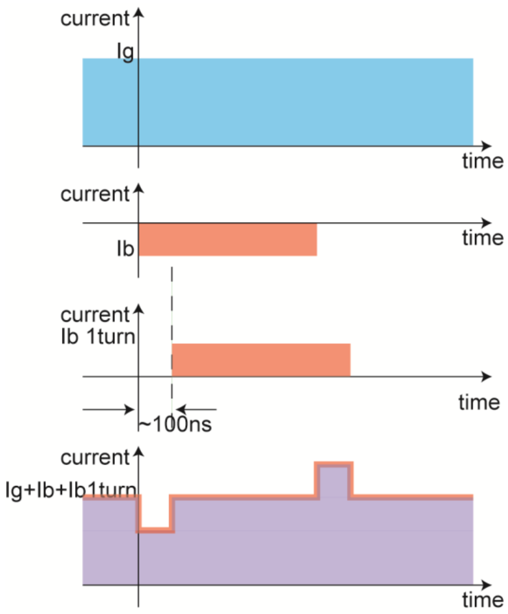
Injector 2&3(Ig=101.5mA, Ib=10mA, beam phase=10deg., IbFF=10mA (-10deg.))
ERL_burstInj2.m



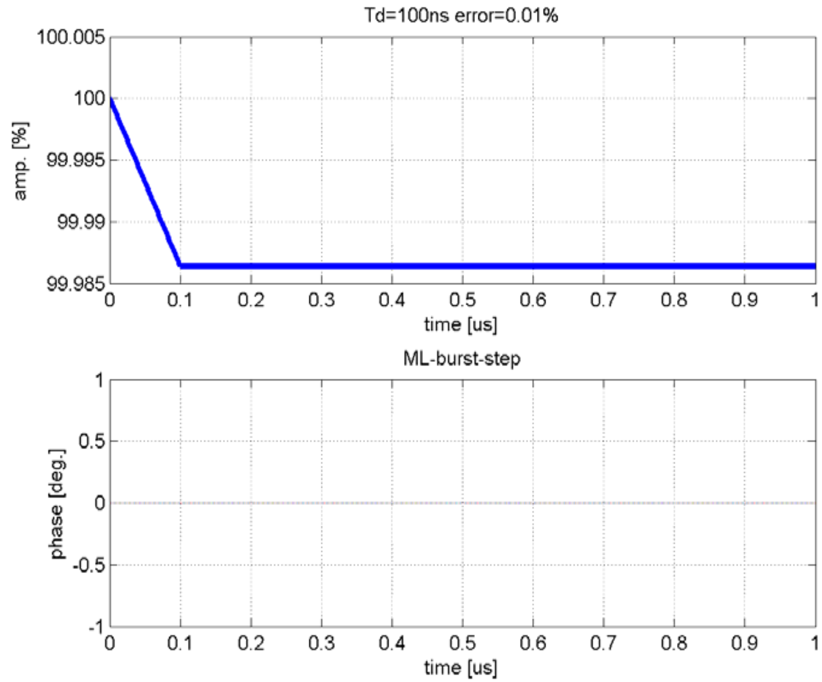
Even if the jitter (between feedforward and beam) is 200ns, amplitude and phase errors are 0.04% and 0.004deg.

Concerning the injector cavities, it will be enough to regulate the timing (between beam and feedforward) to be less than 200ns.

Main linac performance



Rectangular beam



The beam loading effects are only 0.01% for 100ns beam loop delay even at rectangular shape.