RGSのELEGANTへの実装の経過報告*

*Status of Implementation of RGS in ELEGANT

2013年10月22日(火)14時00分
ビームダイナミクスWG打ち合わせ

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  - Integrated loss rate & differential scattering rate
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Motivation

- Implement a routine for residual gas scattering (RGS) into ELEGANT* tracking code

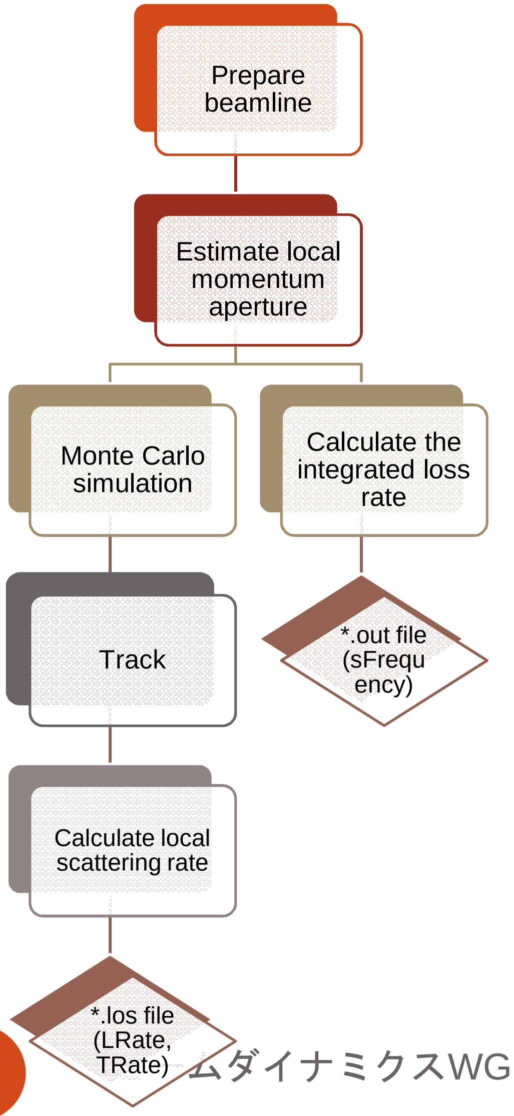
Main idea:

- To use “touschekScatter.c” routine from ELEGANT tracking code as the sample for “RGScatter.c” routine

** M. Borland, "elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation," Advanced Photon Source LS-287, September 2000.*

Step I: touschekScatter.c code analysis

Routine structure



1. Prepare beamline: Insert scattering objects into beamline, upload beamline setup parameters
2. Estimate local momentum aperture: Twiss parameters and bunch parameters are required
3. Calculate the integrated loss rate from Piwinski's formula->sFrequency output
4. Monte Carlo Simulation: Generate scattered electrons using Monte Carlo method
5. Track simulated electrons to the end of the baemline
6. Calculate local scattering rate: Calculate the differential scattering rate from Piwinski's formula->LRate, TRate output
7. Record beam loss information

Step I: touschekScatter.c code analysis

Integrated loss rate calculation

- Integrated loss rate*:

$$R = \frac{dN}{dt} = \int \rho_1 \rho_2 v_{rel} \sigma dV$$

$N_{beam} = Q / e$ – number of electrons in “beam” bunch

ρ_1, ρ_2 – electron density

$v_{rel} = c\beta_{rel}$ – relative velocity of 2 electrons

σ – integrated Moeller scattering cross-section

dV - bunch sub-volume

- Moeller cross-section

$$\frac{d\sigma}{d\Omega} = \left(1-\beta^2\right)\left(1+\frac{1}{\beta^2}\right)^2\left(\frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta}\right) + \\ +\left(1-\beta^2\right)\left(1+\frac{4}{\sin^2\theta}\right)$$

A. Piwinsiki, “The Touschek effect in strong focusing storage rings,” DESY 98-179, Nov. 1998. p. 7 Eq. 22, p. 8 Eq. 28

- Analytic loss rate**:

$$R = \frac{N_{beam}^2 r_e^2 c \beta_x \beta_y}{8\sqrt{\pi}\beta^2\gamma^4\sigma_x^2\sigma_y^2\sigma_s} F\left(\tau_m\right)$$

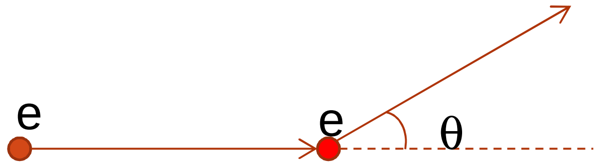
β_x, β_y – beta functions

r_e – radius of electron

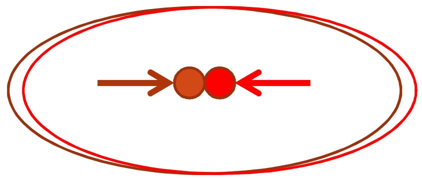
$\sigma_x, \sigma_y, \sigma_s$ – bunch size

$F\left(\tau_m\right)$ – Integral ($\tau_m \to \infty$)

$\tau_m = \beta^2 \left(\Delta p / p\right)_{\min}^2$ – momentum aperture (times β^2)

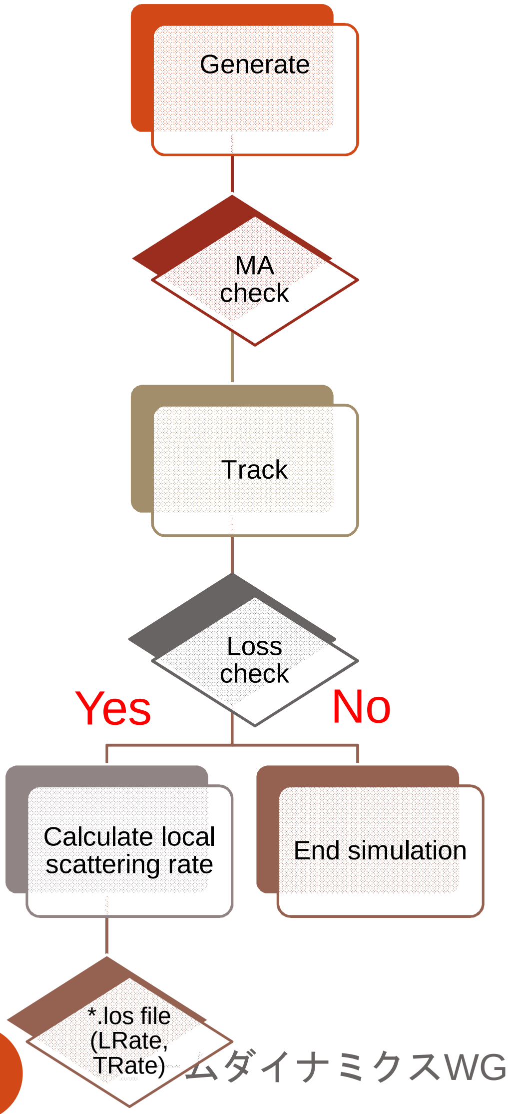


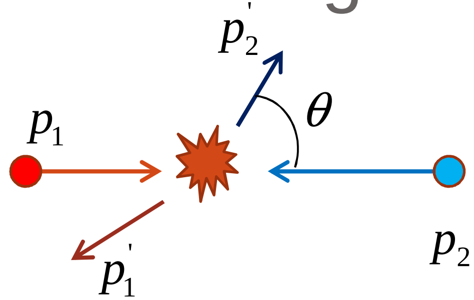
A TS scattering is as if two bunches collide. Note: for each two electrons, the CM frame is different



Step I: touschekScatter.c code analysis

Monte Carlo simulation & differential scattering rate





1. θ_i is generated randomly for each electron track i -> $d\sigma(\theta_i)/d\theta$
2. Check the momentum aperture
   if $\frac{p_1' - p_0}{p_0} > \delta_m$ or $\frac{p_2' - p_0}{p_0} > \delta_m$ ->
   -> tracking; $p_0 = p\_central =$ reference momentum; $\left|\frac{\Delta p}{p}\right| = \delta_m$
3. Calculate differential scattering rate -> LRate, TRate output

$$w = \frac{dN}{dt d\Omega} = \frac{N_e^2 r_e^2 v_{rel}}{\sigma_x \sigma_y \sigma_s \gamma_{CM}^2} \frac{d\sigma}{d\Omega} = N_{beam} v_{rel} \rho_{target} \frac{d\sigma}{d\Omega}$$

β_{CM} – β of 2 particles within a bunch in CM sys.

γ_{CM} – γ of 2 particles within a bunch in CM sys.

N_e – number of electrons in bunch

$\sigma_x, \sigma_y, \sigma_s$ – bunch size

N_{gen} – number of generated particles

$d\sigma/d\Omega$ – differential Moeller cross-section

ρ_{target} - electron density in “target” bunch

Step I: touschekScatter.c code analysis

*.out, *.los output files data

How to get loss rate per second per meter (sFrequency):

1. Check if the scattered electron's momentum is outside of momentum aperture
2. If outside the aperture, transport the electron i originated from element j (position s_j^i) to the loss point (position $s_j^{i'}$)
3. Pick up electrons lost at all scattering elements within s range of 1m step (n, n+1), n=0,1,2,…,100m
4. Calculate loss rate per secons per meter as the sum W_j^i for all selected electrons , where

$$sFrequency_j = \sum_i W_j^i$$

W_j^i - normalized rate for electron track i (TRate)

$$W_j^i = r_j w_j^i / \sum_i w_j^i$$

$r_j = R_j \Delta s_j f / c$ - loss rate per sec at element j

R_j – loss rate (dN/dτ) is calculated with use of Piwinski formula for 1 bunch

Δs_j – thickness of TSO # j

f – frequency

c – speed of light

w_j^i - loss weight for particle i in element j (LRate) w calculated at element # j position for electron track i (N_{gen} is a number of generated particles)

Step II: RGScatter.c code construction

Input parameters & output data

TS

- Required input
  - Twiss parameters from ELEGANT
  - Momentum aperture file from ELEGANT
  - Bunch parameters
    - Charge
    - Maximum energy
    - Repetition
    - Normalized emittance
    - Momentum spread
    - Bunch length
- Output
  - Local scattering rate
  - Integrated beam loss rate

RGS

- Required input
  - Twiss parameters from ELEGANT
  - Bunch parameters
    - Charge
    - Maximum energy
    - Repetition
    - Normalized emittance
    - Momentum spread
    - Bunch length
  - Residual gas parameters
    - Gas pressure
    - Gas component number (up to 10)
    - Component fraction (%)
    - Charge number
    - Mass number
- Output – same!

Step II: RGScatter.c code construction
Physics behind

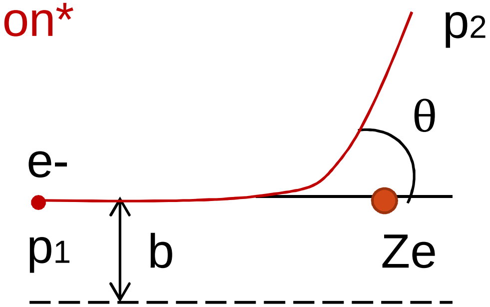
- Differential elastic (Rutherford) scattering cross-section*

Elastic scattering (classical)

$$\frac{d\sigma}{d\Omega}=\frac{Z^2 r_p^2}{4m_e^4 c^4 v^4 \sin^4\left(\frac{\theta}{2}\right)}$$

Elastic scattering (QED)

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma}{2\pi\sin\theta d\theta}=\frac{Z^2 r_p^2 m_e^2\left(1-\beta^2\sin^4\left(\frac{\theta}{2}\right)\right)}{4p^2\beta^2\sin^4\left(\frac{\theta}{2}\right)}$$



$$\vec{P}_1\cdot\vec{P}_2=|\vec{P}|^2\cos\theta=|\vec{P}|^2(1-2\sin^2\frac{\theta}{2})$$

Z - atomic number of nucleus
r_p - classical electron radius
m_e - electron mass
v - electron velocity
θ – electron scattering angle
p - momentum of electron
b - electron velocity / c

- Integrated scattering cross-section

$$\sigma=\int_{\theta_{acc}}^{\pi}\frac{d\sigma}{d\theta}d\theta=\frac{Z^2 r_p^2 m_e^2}{4p^2\gamma^2}\left\{-\frac{2}{\beta^2}\left(\frac{1}{2}-\frac{1}{1-\cos\left(\theta_{acc}\right)}\right)-\log\left(\frac{2}{1-\cos\left(\theta_{acc}\right)}\right)\right\}$$

$\theta_{acc}^{j}=\sqrt{H/\beta_j}$ - transverse angle acceptance at element j

$H=\left(A(s)^2/\beta(s)\right)_{\min}$ - machine acceptance = min aperture over the beamline

* http://www7b.biglobe.ne.jp/~kcy05t/rathef.html

Step II: RGScatter.c code construction

Integrated loss rate & differential scattering rate

- Integrated loss rate*:

$$R = \frac{dN}{dt} = N_{beam} v_{beam} \rho_{target} \sigma$$

$N_{beam} = Q / e$ – number of electrons in bunch

$\rho_{target} = N_A P_{duct} / V_{STP} P_{1atm}$ – residual gas density

$v_{beam} = v_{rel} = v$ – bunch velocity

σ – integrated elastic (Rutherford) scattering cross-section

- Differential scattering rate*:

$$w = \frac{dN}{dt d\Omega} = N_{beam} v_{beam} \rho_{target} \frac{d\sigma}{d\Omega}$$

Scattering rate per bunch per solid angle

** Halzen F., Martin A.D. Quarks and leptons. introductory course in modern particle physics (Wiley, 1984)*

- Gas component mixture:

$$R = \sum_{k=1}^{n} R_k = \sum_{k=1}^{n} N_{beam} v_{beam} \rho_{target,k} \sigma_k$$

f_k – fraction of atom k

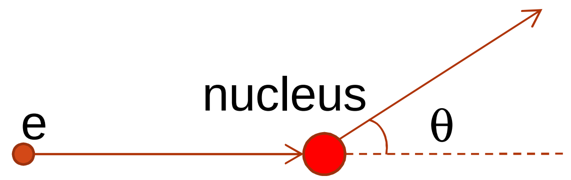
Z_k , A_k – charge and mass number of atom k

N_A – Avogadro constant

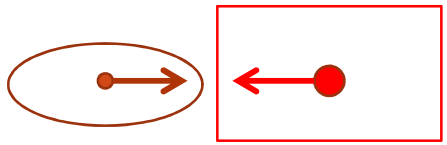
V_{STP} – volume of gas at 27^{o}C, 1 atm

P_{duct} – pressure in a beam duct

P_{1atm} – 1 atm



RGS is a collision between bunch and nucleus with the uniform density in the beam duct



Current result

Input data

Main Parameters*	
Maximum energy	20 MeV
Current	10 mA
Charge per bunch	7.7 pC
Repetition	1.3 GHz
Normalized emittance	1 mm-mrad
Momentum spread	1 x 10-3
Bunch length	2 ps
Gas pressure*	1e-6 Pa
Gas component number**	2
Component fraction	0.5, 0.5
Charge number	12, 16
Mass number	6, 8

*** 中村さんとのディスカッションからの。**

RG - Notepad

File Edit Format View Help

```
        n_steps = 1,
&end

&insert_elements
        name = *,
        type = *,
        s_start = 0.0,
        s_end = 100.0,
        element_def = "RGSO: RGSCATTER",
&end

&rg_scatter
        frequency = 1.3e9,
        charge = 7.7e-12,
        emit_nx = 1e-6,
        emit_ny = 1e-6,
        sigma_dp = 1e-3,
        sigma_s = 6e-4,
        distribution_cutoff[0] = 5*5,
        bunch = %s-%03ld.rgbun,
        loss = %s-%03ld.rglos,
        !distribution = %s-%03ld.dis,
        output=%s-%03ld.rgout,
        !initial = %s-%03ld.ini,
        verbosity=2,
        i_start = 1,
        i_end = 616,
        do_track = 1,
        nbins=100,
        n_simulated = 5000000,
        ignored_portion = 0.01,
        match_position_only = 1,
        overwrite_files=1,
        gas_pressure=1.0e-6,
        n_gas=2,
        gas_fraction[0] = 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0, 0,
        gas_Z[0] = 6, 8, 0, 0, 0, 0, 0, 0, 0, 0,
        gas_A[0] = 12, 16, 0, 0, 0, 0, 0, 0, 0, 0,
&end

&stop &end
```

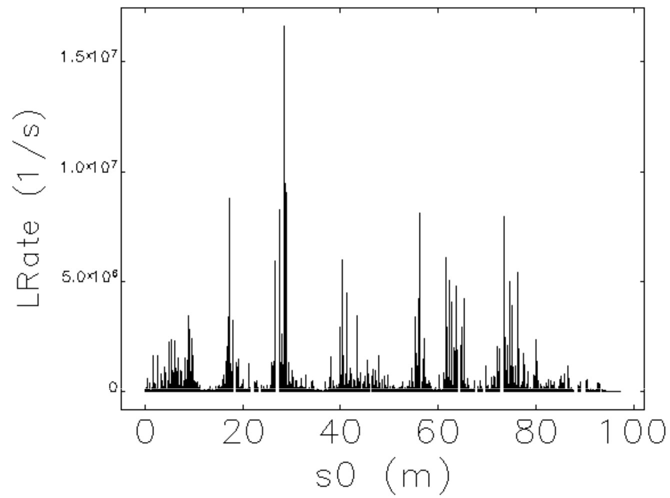
* typical for cERL

** Carbon monoxide (CO)

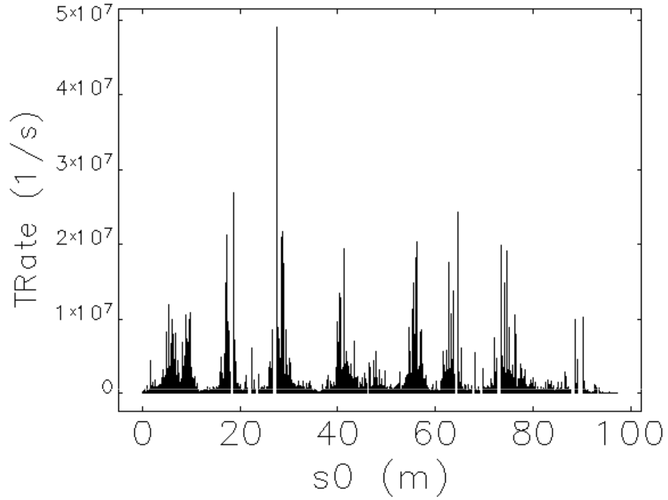
Current result

Beam loss rate

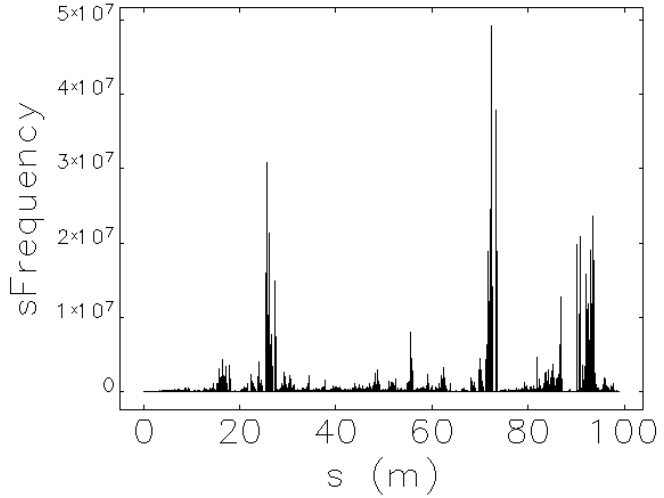
- Normalized scattering rate [particle/second]



- Differential scattering rate [particle/second]



- Integrated loss rate [particle/second*meter]



Loss Rate Comparison	
TS Rate	~ 600-700 pA/m
RGS Rate	~ 70-90 pA/m
RGIS	?

Summary & future work

- RGScatter routine was constructed on the base of touschekScatter.c routine
    - A new routine is completely separated from the sample one
    - First simulation results are reasonable
    - Both of elastic and inelastic scattering can be implemented
- First stage of routine implementation is completed
    - Theoretical estimation
    - Comparison with experimental data
    - Debugging
- The routine for inelastic scattering case was also implemented (RIScatter.c) -> simulation results not completely satisfactory - > debugging

Conclusion
Aknowlegements

- Thanks to Professor Nakamura-san and Assistant Professor Shimada-san for important advises, useful criticism and fruitful discussions during preparation of this results
- Special thanks to Sako-san (J-PARC) for the technical support and for the help with programming code manipulations

Thank you for attention!

Appendix A:
Inelastic scattering cross section*

- **Differential cross section**
  - Photon emission in e + nucleus

$$\left(\frac{d\sigma}{d\varepsilon}\right)_N = \alpha\frac{4Z^2{r_e}^2}{\varepsilon}\left\{\left[\frac{4}{3}\left(1-\frac{\varepsilon}{E}\right)+\left(\frac{\varepsilon}{E}\right)^2\right]\left[\log\left(\frac{183}{Z^{\frac{1}{3}}}\right)\right]+\frac{1}{9}\left(1-\frac{\varepsilon}{E}\right)\right\}$$

  - Photon emission in e + e (bounded in the atom)

$$\left(\frac{d\sigma}{d\varepsilon}\right)_e = \alpha\frac{4Z\ \ {r_e}^2}{\varepsilon}\left\{\left[\frac{4}{3}\left(1-\frac{\varepsilon}{E}\right)+\left(\frac{\varepsilon}{E}\right)^2\right]\left[\log\left(\frac{1194}{Z^{\frac{2}{3}}}\right)\right]+\frac{1}{9}\left(1-\frac{\varepsilon}{E}\right)\right\}$$

- **Integrated cross section**

$$\sigma = \int_{\varepsilon_m}^{E}\left[\left(\frac{d\sigma}{d\varepsilon}\right)_N+\left(\frac{d\sigma}{d\varepsilon}\right)_e\right]\ d\varepsilon$$

$$= \alpha\frac{4Z^2{r_e}^2}{\varepsilon}\left\{\left[\frac{4}{3}\log\frac{E}{\varepsilon_m}-\frac{4}{3E}(E-\varepsilon)+\frac{1}{2E^2}(E^2-\varepsilon^2)\ \right]F(Z)+\frac{1}{9}\left[\log\left(\frac{E}{\varepsilon}\right)-\frac{E-\varepsilon}{E}\right]Z(Z\right.$$

$$\left.+1)\right\}$$

$$F(Z) = Z^2\log\frac{183}{Z^{\frac{1}{3}}}+Z\log\frac{1194}{Z^{\frac{2}{3}}}$$

** H. W. KOCH and J. W. MOTZ Bremsstrahlung Cross-Section Formulas and Related Data Rev. Mod. Phys. 31, 920–955 (1959)*