

トシエック散乱による ビームロスの計算結果修正*

*Beam Loss Due To Touschek Scattering. Simulation Results Corrections

2013年10月22日(火)14時00分
ビームダイナミクスWG打ち合わせ


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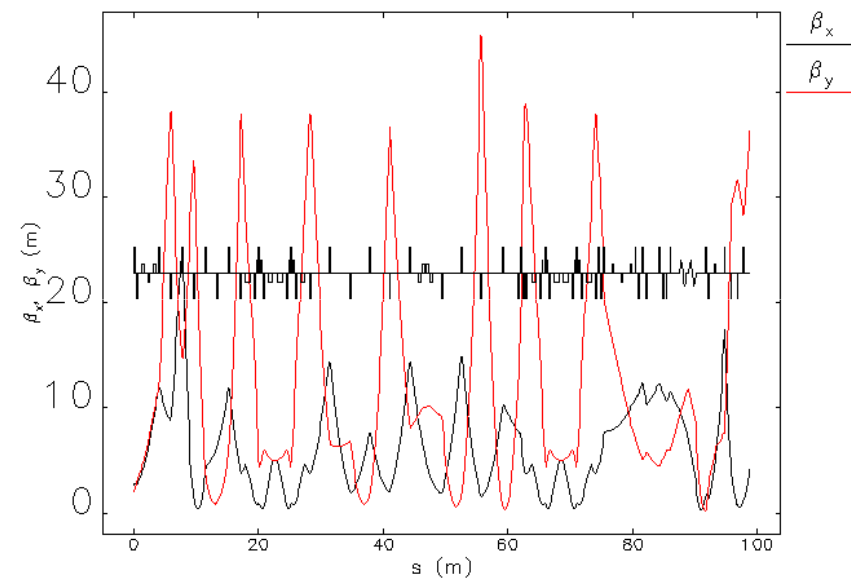
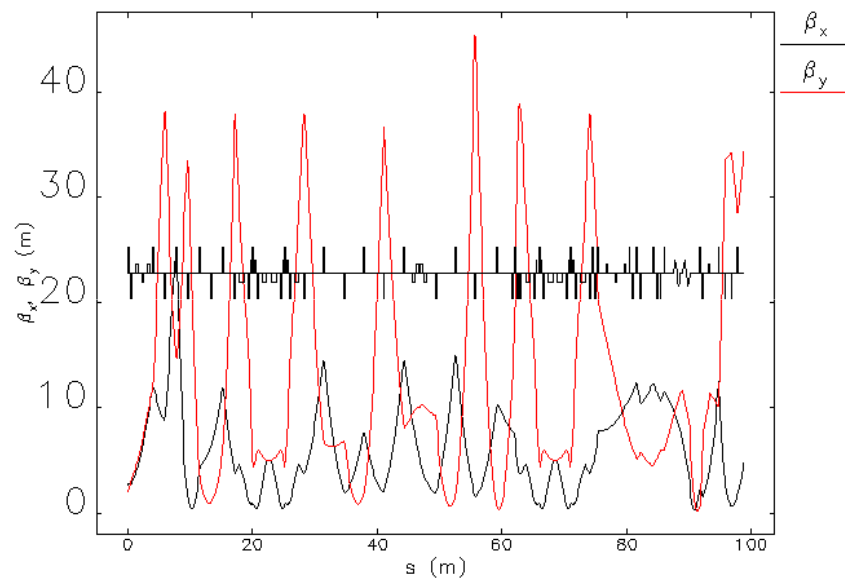
Outline

- Input data
 - Lattice & optics
 - Beam parameters
- Results
 - Beam loss comparison
 - 35.5MeV vs 20MeV
 - Bunch length, momentum spread & normalized emittance influence
 - Piwinski's loss rate estimation
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- Summary

Input Data

Lattice & Optics

normal-20121122.new (E_{max} = 35.5MeV)  E_{max} = 20MeV



Maximum energy was modified → Twiss parameters had changed →
Beam loss position had changed

* 島田さんからのlatticeファイルを使用

see also: <http://pfconrg07.kek.jp:8082/trac/cERL-Lattice/wiki>

Input Data

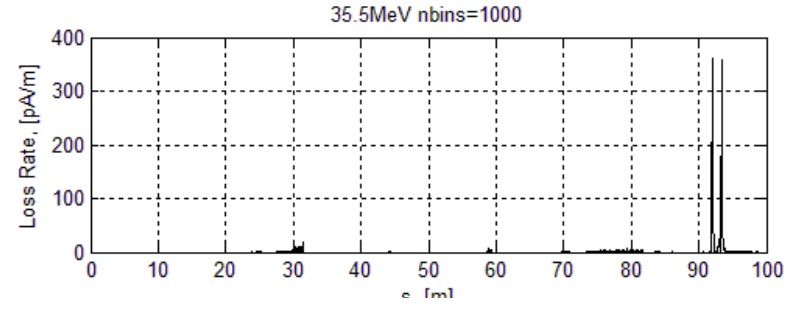
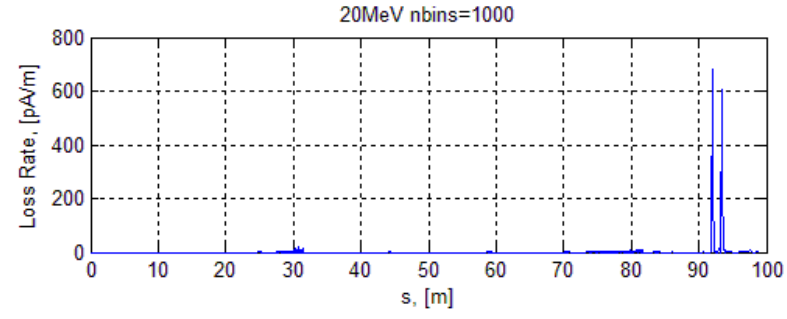
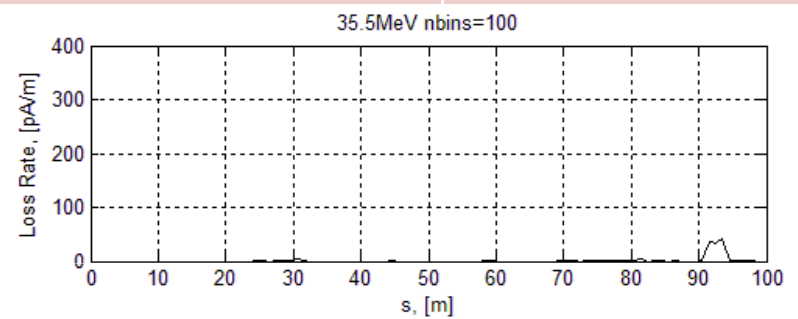
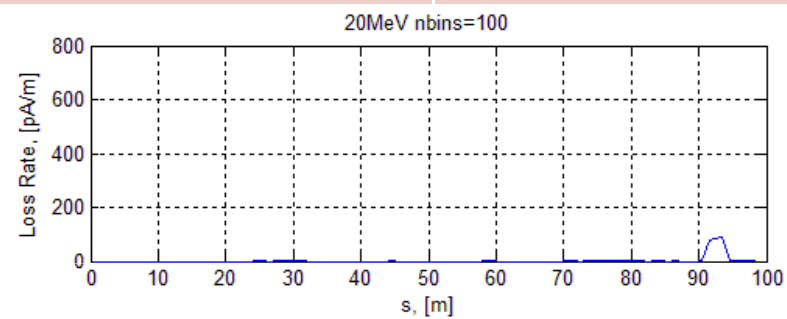
Beam parameters

Main Parameters*	cERL	Simulation	
		Old	New
Maximum energy	20 MeV	35.5 MeV	20 MeV
Current	10 – 100 mA	10 mA	10 mA
Charge per bunch	7.7 – 77 pC	7.7 pC	7.7 pC
Repetition	1.3 GHz	1.3 GHz	1.3 GHz
Normalized emittance	0.1 – 1.0 mm-mrad	0.3 mm-mrad	1 mm-mrad
Momentum spread	$< 3 \times 10^{-4}$	3×10^{-4} eV	1×10^{-3} eV
Bunch length	1 – 3 ps	3 ps	2 ps

* K. Harada, M. Shimada, A. Ueda , Y. Kobayashi, T. Miyajima, R. Hajima , N. Nakamura / Lattice design of the compact ERL for first commissioning; Proceedings of the 7th Annual Meeting of Particle Accelerator Society of Japan (August 4-6, 2010, Himeji, Japan)

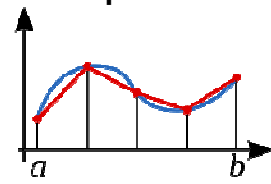
Results: Beam Loss Comparison for $E_{max}=35.5\text{MeV}$ and $E_{max}=20\text{MeV}$

20MeV+100bins	20MeV+1000bins	35.5MeV+100bins	35.5MeV+1000bins
$\int = 284.8597$	$\int = 284.8323$	$\int = 136.6370$	$\int = 138.2992$



Total volume of the beam losses is calculated with use of Trapezoidal rule

Beam Loss Ratio
(20MeV vs 35.5MeV)
is 2.05953



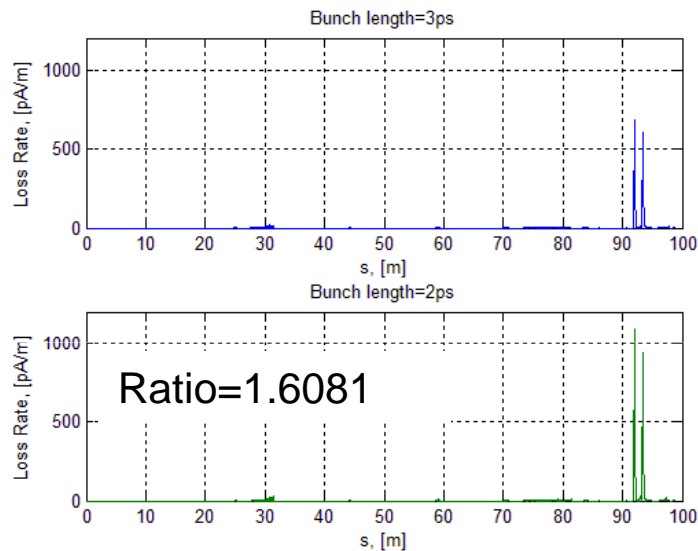
$$\int_a^b f(x) dx = \frac{f(a) + f(b)}{2} (b - a) + E(f),$$

$$E(f) = -\frac{f''(\xi)}{12} (b - a)^3$$

或25年10月22日

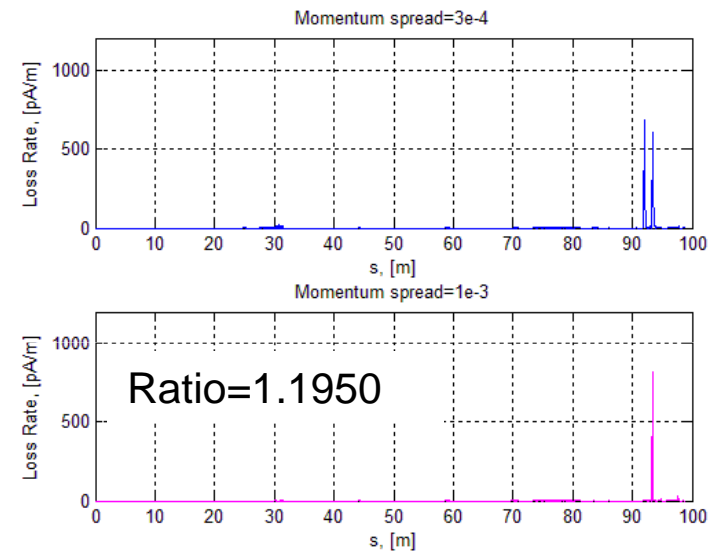
Results: Beam Loss Comparison for $E_{\max}=20\text{MeV}$ and different beam parameters

Bunch length 3ps vs 2ps



- When the bunch density is growing the probability of Touschek scattering is increasing -> loss rate is growing

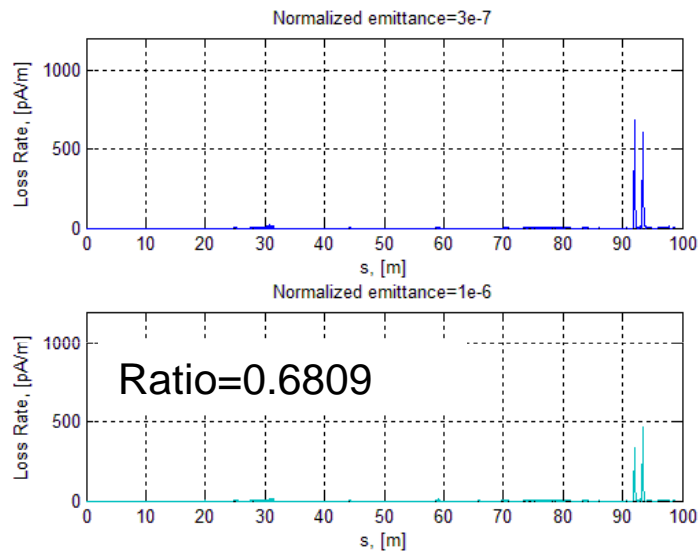
Momentum spread $3e-4$ vs $1e-3$



- When the momentum spread is growing the number of particles with momentum exceeding the momentum aperture is increasing -> loss rate is growing

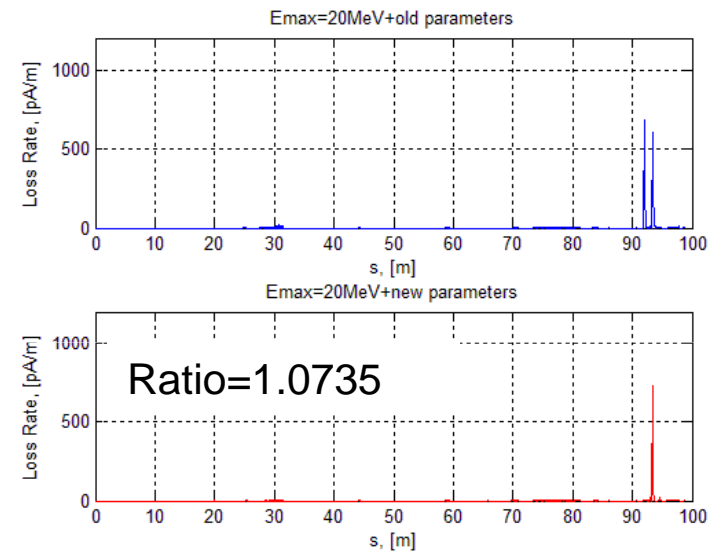
Results: Beam Loss Comparison for $E_{max}=20\text{MeV}$ and different beam parameters

Normalized emittance $3e-7$ vs $1e-6$



- When the normalized emittance is growing the bunch density is decreasing -> loss rate is decreasing

20MeV old param. vs new param.



- When the total influence of all 3 parameters (bunch length, momentum spread, normalized emittance) is summarized -> loss rate is slightly increasing

Results: Piwinski Loss Rate

General equation

Input parameters

r_p =radius of the particle=2.8179e-15 [m]

c =3e8 [m/sec]

$\beta_{x,y}$ =betatron function (from init.twi file)

N_p =number of particles per bunch=
=charge in bunch/electron charge

β =0.99967 (for E_{max} =20MeV)

γ =39.12894 (for E_{max} =20MeV)

m_e =0.511[MeV]

$D_{x,y}$ =dispersion function (from init.twi file)

$\sigma_{x\beta}, \sigma_{y\beta}$ =horizontal & vertical betatron width
= $\sqrt{(\beta_{x,y} * emit_{x,y})}$

$emit_{x,y}$ =normalized emittance=1e-6 [mm-
mrad]

σ_s =rms bunch length=6e-4 [m]

σ_p =relative momentum spread=1e-3

$$R [particles/s] = \frac{r_p^2 c \beta_x \beta_y \sigma_h N_p^2}{8 \sqrt{\pi} \beta^2 \gamma^4 \sigma_{x\beta}^2 \sigma_{y\beta}^2 \sigma_s \sigma_p} F(\tau_m, B_1, B_2)$$

Momentum Aperture:

$$\tau_m = \beta^2 \delta_m^2 = \beta^2 \left(\frac{\Delta p_m}{p} \right)^2$$

$$\sigma_h = \frac{\sigma_{x\beta} \sigma_{y\beta} \sigma_p}{\sqrt{\tilde{\sigma}_x^2 \sigma_{y\beta}^2 + \tilde{\sigma}_y^2 \sigma_{x\beta}^2 - \sigma_{x\beta}^2 \sigma_{y\beta}^2}}$$

$$F = \int_{\tau_m}^{\infty} \left(\left(2 + \frac{1}{\tau} \right)^2 \left(\frac{\tau}{\tau_m} - 1 \right) + 1 - \frac{\sqrt{1+\tau}}{\sqrt{\tau/\tau_m}} - \frac{1}{2\tau} \left(4 + \frac{1}{\tau} \right) \ln \frac{\tau/\tau_m}{1+\tau} \right) e^{-B_1 \tau} I_0(B_2 \tau) \frac{\sqrt{\tau} d\tau}{\sqrt{1+\tau}}$$

$$B_1 = \frac{\beta_x^2}{2 \beta^2 \gamma^2 \sigma_{x\beta}^2} \left(1 - \frac{\sigma_h^2 \tilde{D}_x^2}{\sigma_{x\beta}^2} \right) + \frac{\beta_y^2}{2 \beta^2 \gamma^2 \sigma_{y\beta}^2} \left(1 - \frac{\sigma_h^2 \tilde{D}_y^2}{\sigma_{y\beta}^2} \right)$$

$$\tilde{\sigma}_{x,y}^2 = \sigma_{x\beta,y\beta}^2 + \sigma_p^2 (D_{x,y}^2 + \tilde{D}_{x,y}^2)$$

$$\tilde{D}_{x,y} = \alpha_{x,y} D_{x,y} + \beta_{x,y} D'_{x,y}$$

$$B_2 = B_1^2 - \frac{\beta_x^2 \beta_y^2 \sigma_h^2}{\beta^4 \gamma^4 \sigma_{x\beta}^4 \sigma_{y\beta}^4 \sigma_p^2} (\sigma_x^2 \sigma_y^2 - \sigma_p^4 D_x^2 D_y^2)$$

Results: Piwinski Loss Rate* Estimation

Parameters obtained from
MATLAB estimation
(mean value)

$\sigma_h=9.455387418879311e-04$

$N_p=48125000$

$\beta_x= 5.900141513614267$

$\beta_y= 12.912173749594803$

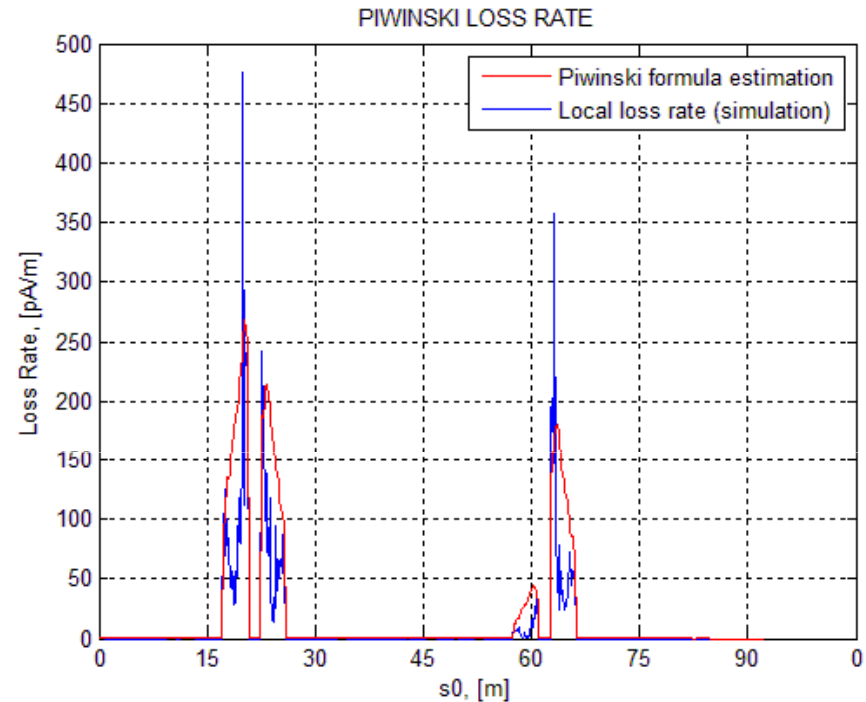
$\sigma_{x\beta}=0.002264489534041$

$\sigma_{y\beta}=0.003288459226620$

$\min(\tau_m)=0.000020057$

$B1= 6.102909035031659e+03$

$B2= 3.210975722167509e+03$



Good agreement with the local loss rate for
 $E_{max}=20\text{MeV}$ +old parameters simulation

A. Piwinski, "The Touschek effect in strong focusing storage rings," DESY 98-179, Nov. 1998. p. 7 Eq. 22, p. 8 Eq. 28

Summary

- Because of the shift from $E_{\max}=35.5\text{MeV}$ to $E_{\max}=20\text{MeV}$ optics beam loss due to Touschek scattering increased significantly
- Toschek scattering rate depends on the bunch parameters in a very complicated way
 - Loss rate is growing due to decreasing the bunch length
 - Loss rate is growing due to increasing the momentum spread
 - Loss rate is growing due to decreasing normalized emittance
- Theoretical estimation by the Piwinski's formula for the beam loss rate is in a good agreement with simulation results -> comparison with experimental data