散乱によるビームロス*

*Beam loss due to the scattering

2014年1月28日(火)14時00分
ビームダイナミクスWG打ち合わせ

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Outline

- Beam loss studies at Compact-ERL
    - Motivation
    - Lattice & Optics
  - Touschek effect
  - Intra-Beam Scattering
  - Residual Gas Scattering (elastic, inelastic)
    - Summary & Outlook

Beam loss studies at Compact ERL

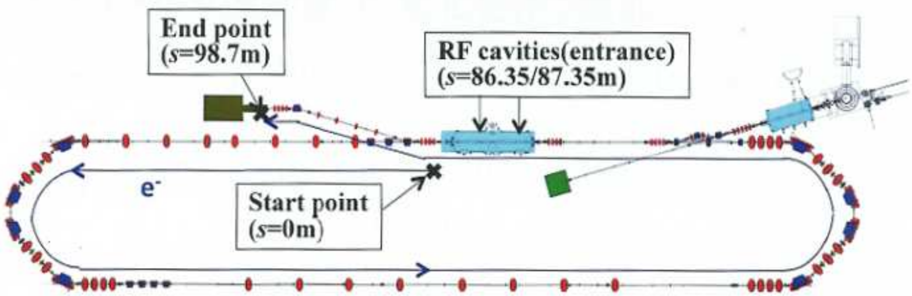
Motivation

3 of the beam loss mechanisms include into ERL beam loss:

1. Touschek Effect (TS) and Intra-Beam Scattering (IBS),
2. Residual Gas Scattering (RGS): elastic (RES) and inelastic (RIS)),
3. Field emission (FE).

- I performed the simulations for TS, IBS, RES and RIS.
- I used existing and modified ELEGANT routine to perform the simulations.
- I also developed a MATLAB data analysis algorithm to handle the large amount of information that is produced by the program.
- Then the data obtained then are compared with the theoretical estimation to verify the accuracy of the simulation.

Beam parameters*	Simulation	cERL
Maximum energy	20 MeV	20 MeV
Total beam current	10 mA	10 – 100 mA
Repetition	1.3 GHz	1.3 GHz
Charge per bunch	7.7 pC	7.7 – 77 pC
Normalized beam emittance	1 mm·mrad	0.1 – 1.0 mm·mrad
Rms momentum spread	$1\cdot10^{-3}$	$< 3\cdot10^{-4}$
Bunch length	2 ps	1 – 3 ps



* S. Sakanaka, IPAC'13, 2013.

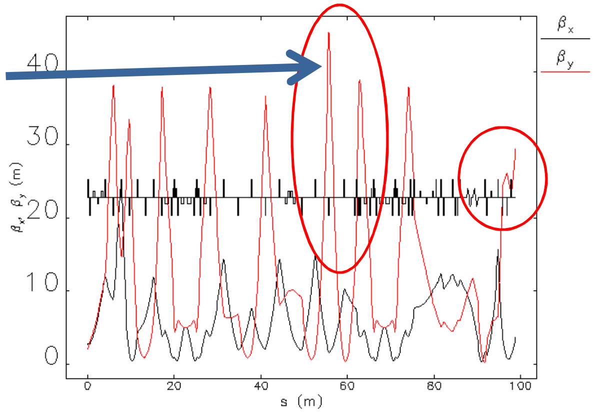
Beam loss studies at Compact ERL

Lattice & Optics*

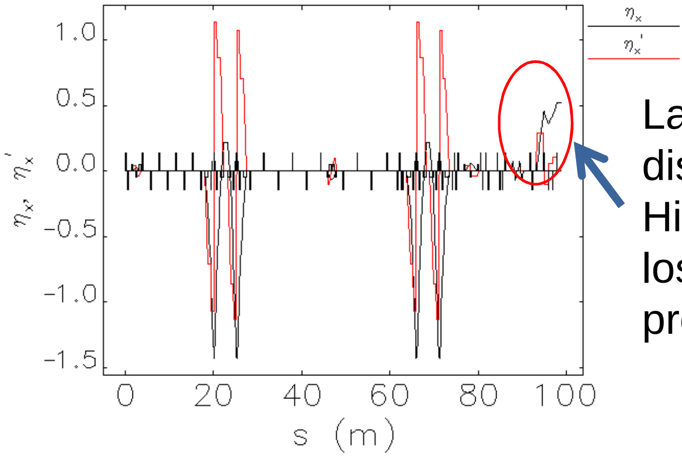
* M. Shimada, ERL09, 2009.

Large betatron amplitude
Higher beam loss probability

Betatron function

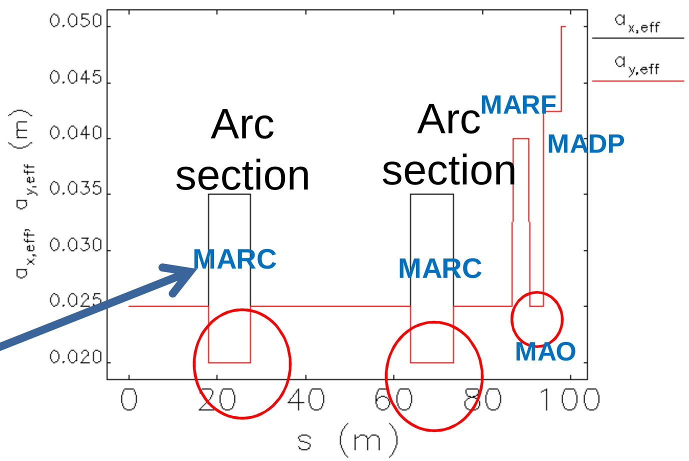


Dispersion



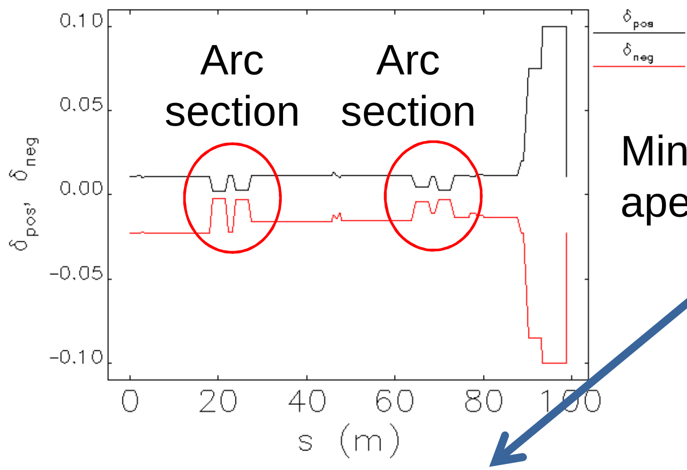
Large dispersion
Higher beam loss probability

Transverse Aperture



Collimators in the lattice

Momentum Aperture



Minima of aperture

Local minima of aperture → **Large beam loss probability?**

Touschek effect
Physics

The Touschek effect is large angle Coulomb collisions in an electron bunch that lead to momentum transfers from the transverse into the longitudinal directions

- Touschek scattering rate*:

$$R\left[\frac{particles}{\sec}\right]=\frac{r_p^2 c\beta_x\beta_y\sigma_h N_p^2}{8\sqrt{\pi}\beta^2\gamma^4\sigma_{x\beta}^2\sigma_{y\beta}^2\sigma_s\sigma_p}\times F\left(\tau_m,B_1,B_2\right)$$

* A. Piwinsiki, DESY, 1998.

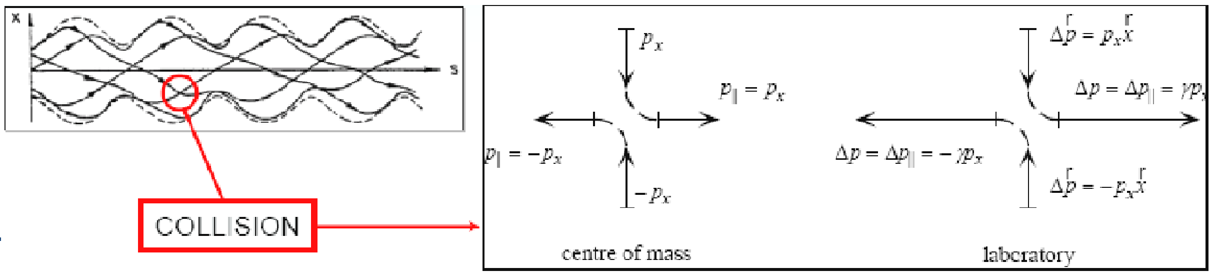
Non-dispersive section (dispersion function=0)

No betatron oscillation occur due to momentum changes in TS. Mainly loss occurs when position deviation due to dispersion at downstream dispersive section exceeds the chamber aperture.

Dispersive section

Betatron oscillation amplitude changes due to position deviation from dispersion.
Loss occurs when deviation due to the beta oscillations exceeds the chamber aperture.

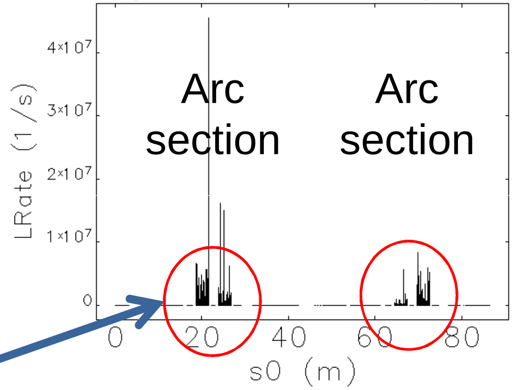
Loss mechanism



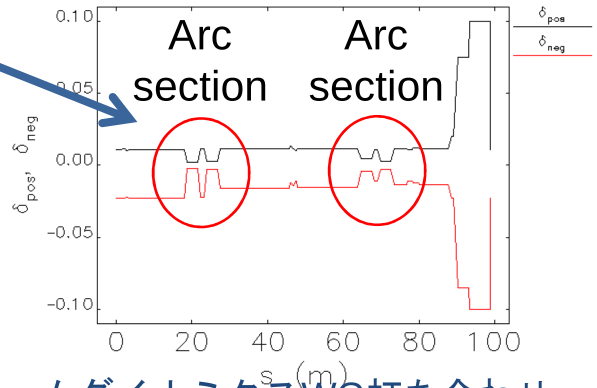
Touschek effect
Result

- Maximum loss rate peak of 21 pA/m is observed at s = 93m where dispersion is large due to bending magnets and transverse aperture is small (2.5cm x 2.5cm collimator in the Lattice).
- The origins of the losses are Touschek scatterings in the two arc sections

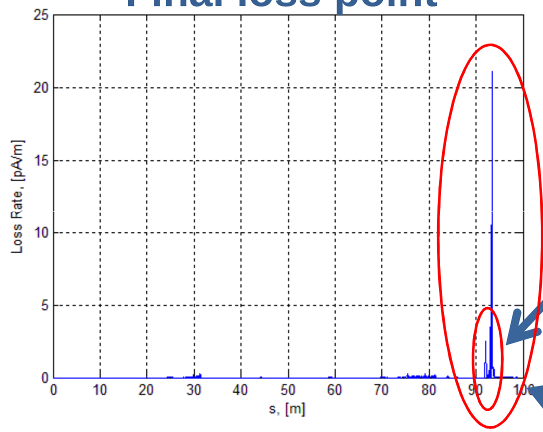
Original scattering point



Lost electrons originate at positions with smallest momentum apertures

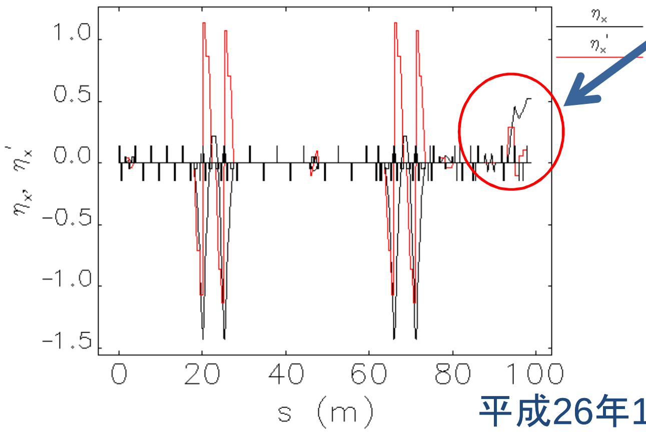


Final loss point



Local minimum of transverse aperture

Dispersion



Touschek effect

Theoretical estimation

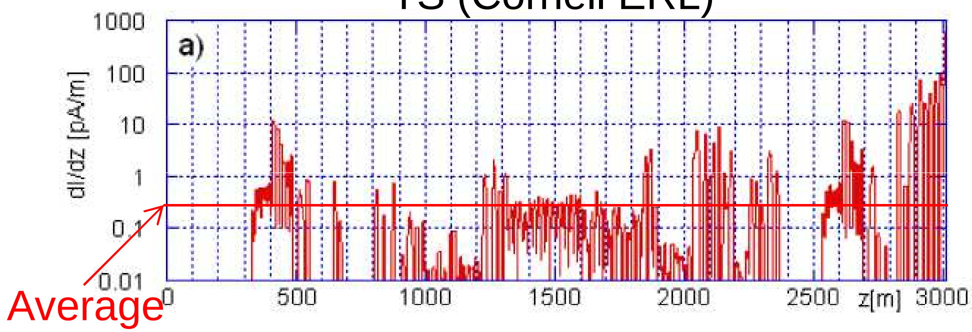
	TS (cERL)	TS (Cornell)
c	3.00E+08	3.00E+08
r0	2.82E-15	2.82E-15
Nb	4.80E+07	4.81E+08
gamma	39.21	9803.92
Vb	6.82E-09	8.18E-12
ex	2.55E-08	3.06E-11
ey	2.55E-08	3.06E-11
bx	10	10
by	10	10
sx	50.49E-5	1.74929E-05
sy	50.49E-5	1.74929E-05
sz	6.00E-04	6.00E-04
sigmax'	5.05E-05	1.74929E-06
MA	0.01	0.01
epsilon	25.50	0.34
Cepsilon	-3.14	-1.27
dN/dt	1.47E+07	9.21E+05
Rate(pA/m)	1.11E-1	6.39E-01

Blue=assumed

C. J. Bochetta, CAS, 2003.

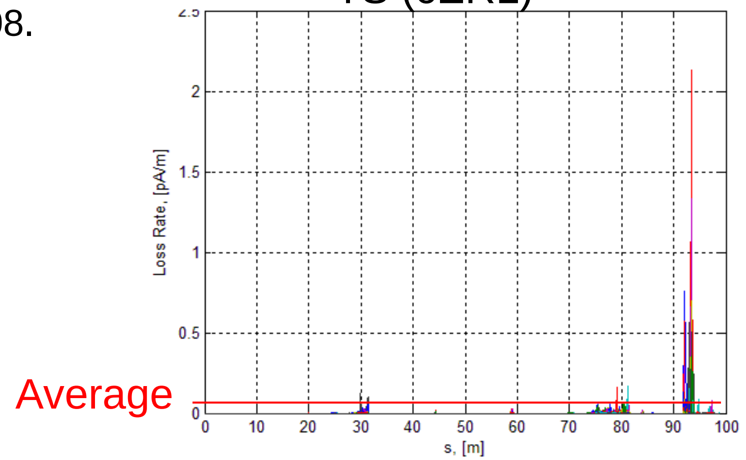
$$\frac{dN}{dt} = -\frac{\sqrt{\pi}\, r_0^2 N_b^2 C(\varepsilon)}{\gamma^3 V_b \sigma_x' \left(\frac{\Delta p}{p}\right)^2},$$

TS (Cornell ERL)



A. B. Temnykh, EPAC,2008.

TS (cERL)



Intra-Beam Scattering
IBS emittance growth rate simulation

Intra-Beam Scattering is a multiple small-angle Coulomb scattering inside the beam. I used a simple ELEGANT routine to examine how the IBS effect impacts the emittance growth. The calculation is based on Bjorken - Mitingwa formula.

- IBS emittance growth rate estimation formula*:

$$\frac{1}{\tau} = \frac{r_p^2 Nc}{\gamma^2 \varepsilon_x \varepsilon_y \sigma_s \sigma_p} \times K_n \Lambda_n \Lambda_c.$$

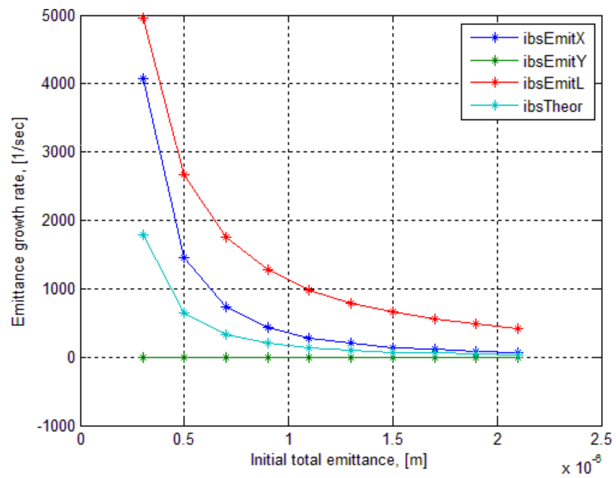
$$K_n = \Lambda_n = 1, \quad \Lambda_c = 10 \sim 15$$

is the Coulomb logarithm for typical electron beams

$$\varepsilon = 0.3 \sim 2.1 \quad mm \cdot mrad$$

For the typical cERL bunch parameters the transverse emittance growth rate is about 450 s^{-1}.

Simulated emittance growth rates for horizontal (“ibsEmitX”), vertical (“ibsEmitY”), and longitudinal (“ibsEmitL”) directions, and theoretical estimation curve (“ibsTheor”).



	cERL
N	5.00E+06
gamma	39.21568627
enx(m rad)	1.00E-06
eny(m rad)	1.00E-06
dp/p	1.00E-03
ds(m)	6.00E-04
F	2.13E+23
1/t	9.57E-06
t(sec)	104442.7122
t(hour)	29.0118645
dt	3.33333E-07
Kn	1
Lambda_n	1
Lambda_c	12.5
re(m)	2.82E-15
c (m/sec)	3.00E+08
1/t(1/s)	**1.61E+02**

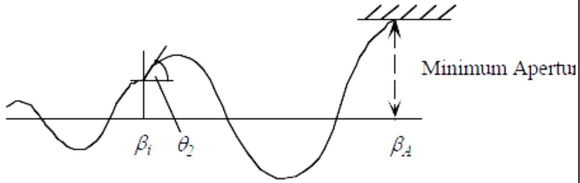
- Only very large IBS growth rates make noticeable the distribution change, because bunch spends in ERL very short time.
- The IBS will cause the significant effect on beam distribution in ERLs in the case of very long transport of high-brightness beams at low energies.

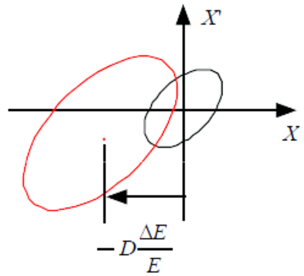
* A. V. Fedotov, KEK, 2011.

Residual Gas Scattering

Physics

C. J. Bocchetta, CAS, 2003.





The interactions between the beam particles and the residual gas atoms/molecules (RGS) may degrade the beam quality and can cause the beam losses. There are two principally different effects: elastic scattering and inelastic scattering.

1. In the elastic scattering, the bunch particles are transversally deflected and its betatron oscillation amplitudes are increased. If the amplitude is large enough to exceed the transverse aperture of the accelerator, the particles are lost.
2. In the case of inelastic scattering the energy of particles is reduced due to Bremsstrahlung on gas nuclei, when the electron is deflected by the residual gas nucleus and it emits a photon. Another way is excitation of a gas atom due to direct energy transfer from the electron to the residual gas atom. The beam particles are lost because the energy is beyond the acceptance of the beam line.

- Differential scattering rate*:

$$w = \frac{dN}{dt d\Omega} = N_{beam} v_{beam} \rho_{target} \frac{d\sigma}{d\Omega}$$

Loss rate per bunch per solid angle (elastic) or per energy (inelastic)

- Scattering rate:

$$R = \frac{dN}{dt} = N_{beam} v_{beam} \rho_{target} \sigma$$

N_{beam} – number incident particles

ρ_{target} – residual gas density

v_{beam} – relative velocity between beam" and "target"

σ – integrated elastic (Rutherford) or inelastic scattering cross-section

- Lost particles number per unit of longitudinal distance:

$$\frac{dN}{dt} = \frac{c}{f} \frac{dN}{ds}$$

f – the repetition frequency (1.3 GHz for cERL)

Residual Gas Scattering
Physics of elastic scattering

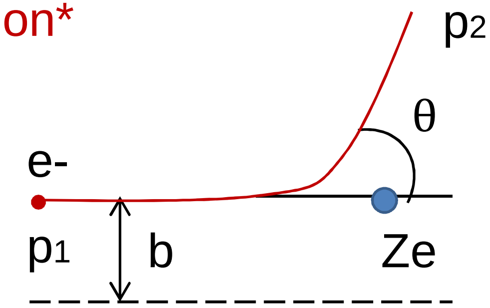
- Differential elastic (Rutherford) scattering cross-section*

Elastic scattering (classical)

$$\frac{d\sigma}{d\Omega}=\frac{Z^2 r_p^2}{4 m_e^4 c^4 v^4 \sin^4\left(\frac{\theta}{2}\right)}$$

Elastic scattering (QED)

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma}{2\pi\sin\theta d\theta}=\frac{Z^2 r_p^2 m_e^2\left(1-\beta^2\sin^4\left(\frac{\theta}{2}\right)\right)}{4p^2\beta^2\sin^4\left(\frac{\theta}{2}\right)}$$



$$\vec{P}_1\cdot\vec{P}_2=|\vec{P}|^2\cos\theta=|\vec{P}|^2(1-2\sin^2\frac{\theta}{2})$$

Z - atomic number of nucleus
r_p - classical electron radius
m_e - electron mass
v - electron velocity
θ – electron scattering angle
p - momentum of electron
b - electron velocity / c

- Integrated scattering cross-section

$$\sigma=\int_{\theta_{acc}}^{\pi}\frac{d\sigma}{d\theta}d\theta=\frac{Z^2 r_p^2 m_e^2}{4p^2\gamma^2}\left\{-\frac{2}{\beta^2}\left(\frac{1}{2}-\frac{1}{1-\cos\left(\theta_{acc}\right)}\right)-\log\left(\frac{2}{1-\cos\left(\theta_{acc}\right)}\right)\right\}$$

$\theta_{acc}^j=\sqrt{H/\beta_j}$ - transverse angle acceptance at element j

$H=\left(A(s)^2/\beta(s)\right)_{\min}$ - machine acceptance = min aperture over the beamline

* http://www7b.biglobe.ne.jp/~kcy05t/rathef.html

Residual Gas Scattering
Physics of inelastic scattering

- Differential inelastic scattering cross-section

$$\frac{d\sigma}{d\varepsilon} = \frac{4\alpha {r_p}^2}{d\varepsilon}\left(\frac{4}{3}\left(1-\frac{\varepsilon}{E}\right)+\frac{\varepsilon^2}{E^2}\right)F(Z) + Z(Z+1)\left(\frac{1}{9}\left(1-\frac{\varepsilon}{E}\right)\right)$$

$$F(Z) = Z^2\left(\log(183) - \frac{1}{3}\log(Z)\right) + Z\left(\log(1194) - \frac{2}{3}\log(Z)\right)$$

Bremsstrahlung **Gas atom excitation**

- Integrated inelastic scattering cross-section

$$\sigma = \int_{dE_{min}}^{dE_{max}} \frac{d\sigma}{d\varepsilon} d\varepsilon = 4\alpha {r_p}^2\left(\frac{4}{3}\log\left(\frac{dE_{max}}{dE_{min}}\right) - \frac{4}{3}\frac{dE_{max}-dE_{min}}{E} + \frac{{dE_{max}}^2 - {dE_{min}}^2}{2E^2}\right)F(Z)$$
$$+Z(Z+1)\left(\frac{1}{9}\left(\log\left(\frac{dE_{max}}{dE_{min}}\right) - \frac{dE_{max}-dE_{min}}{E}\right)\right)$$

Z - atomic number of nucleus
r_p - classical electron radius
E - electron energy
de – energy loss
dE_{max} – largest energy loss
dE_{min} – lowest energy loss

Residual Gas Scattering

Code construction

- Gas component mixture:

$$R = \sum_{i=1}^{N_{gas}} \sigma^i N_{beam} \rho^i_{target} \upsilon.$$

$$\rho^i_{target} = P^i \frac{P^i N_A}{P_{1atm} V_{std}},$$

P^i – the partial pressure of the gas component i

$P_{1atm} = 101325$ (Pa) is the standard atmosphere

$N_A = 6.0221 \times 10^{23}$ (mol^{-1}) is the Avogadro's constant

$V_{std} = 22.414$ (L/mol) is the gas molar volume at the standard temperature and pressure

Residual gas parameters (CO, carbon monoxide) *	
Gas pressure	10^{-6} Pa
Gas component number	2
Component fraction	0.5 and 0.5
Charge number	6 and 8 (C and O)
Mass number	12 and 16

RG - Notepad

File Edit Format View Help

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&end

&insert_elements
        name = *,
        type = *,
        s_start = 0.0,
        s_end = 100.0,
        element_def = "RGSO: RGSCATTER",
&end

&rg_scatter
        frequency = 1.3e9,
        charge = 7.7e-12,
        emit_nx = 1e-6,
        emit_ny = 1e-6,
        sigma_dp = 1e-3,
        sigma_s = 6e-4,
        distribution_cutoff[0] = 5*5,
        bunch = %s-%03ld.rgbun,
        loss = %s-%03ld.rglos,
        !distribution = %s-%03ld.dis,
        output=%s-%03ld.rgout,
        !initial = %s-%03ld.ini,
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        i_end = 616,
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        overwrite_files=1,
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        gas_fraction[0] = 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0, 0,
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        gas_A[0] = 12, 16, 0, 0, 0, 0, 0, 0, 0, 0,
&end

&stop &end
```

* N. Nakamura, private communications

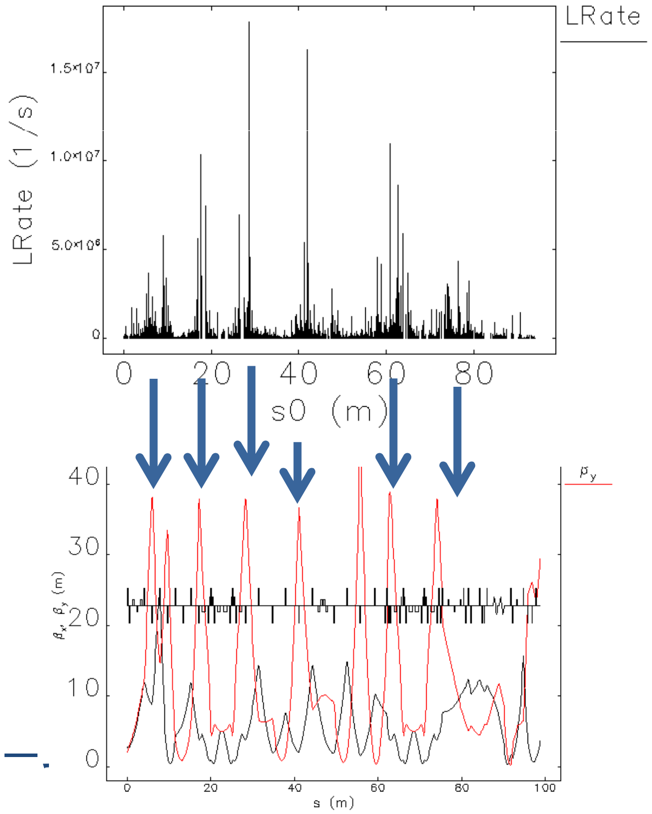
Residual Gas Scattering

Results (I)

- Scattering points are situated in the vacuum chambers
- Beam loss currents due to RGS are small, however, one should concern them as a possible source of irradiation because these processes typically occur in the vertical plane when the amplitude is increased by betatron oscillations.

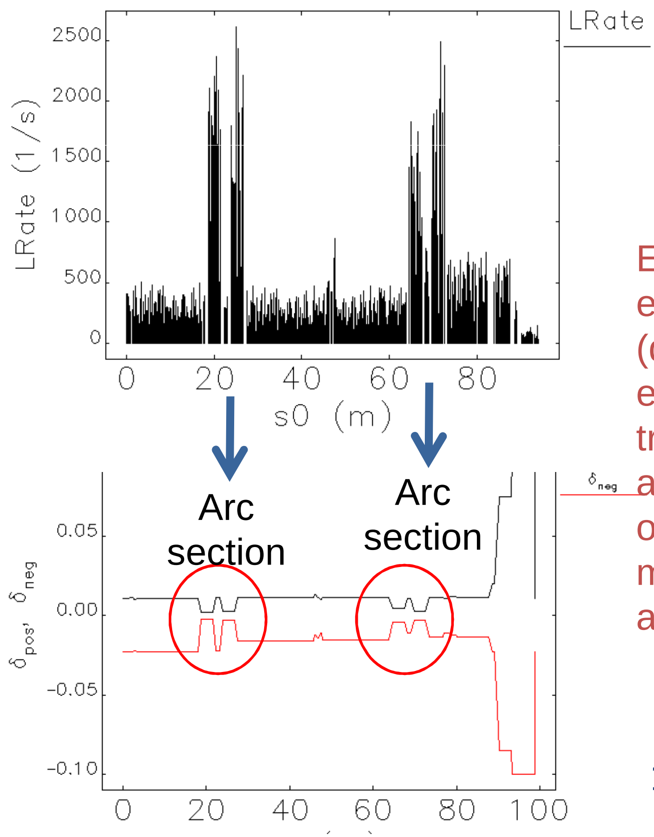
Original scattering points

RES



Circulating electrons are deflected by gas nuclei resulting in an increase of the betatron amplitudes.

RIS



Electron with energy loss (dE/E) is lost either at the transverse aperture limit or at the momentum aperture limit.

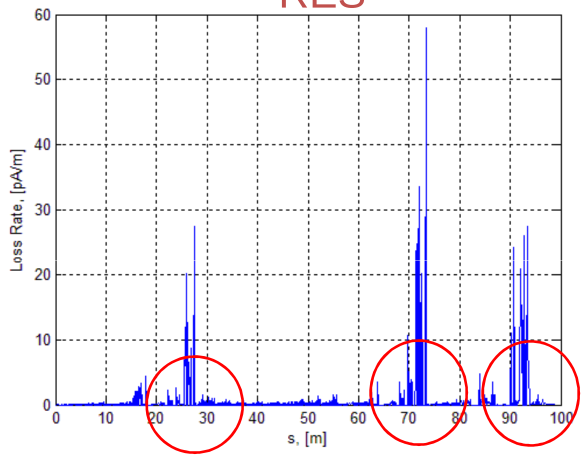
Residual Gas Scattering

Results (II)

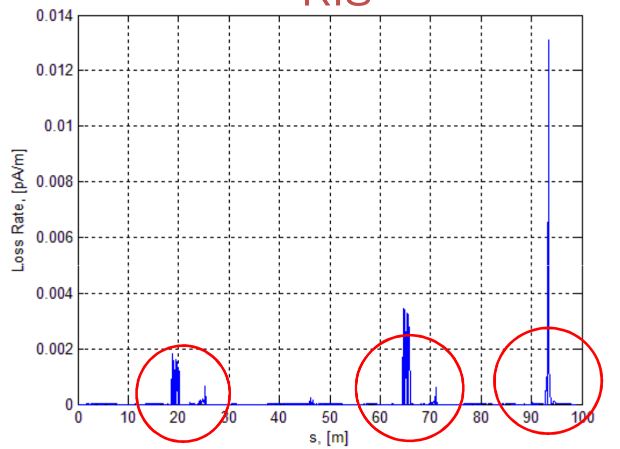
- Thus, I expect the peak loss current to be 58 pA/m for RES and $1.3 \cdot 10^{-2}$ pA/m for RIS; and the average beam loss current due to RES to be 0.76 pA/m and due to RIS to be $5.9 \cdot 10^{-5}$ pA/m.

Final loss points

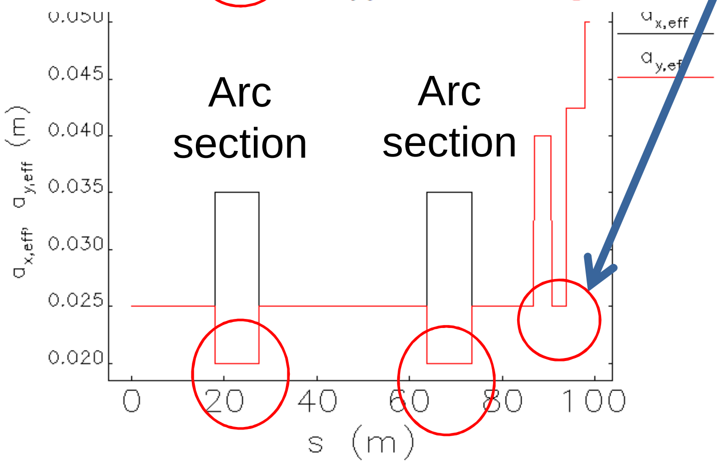
RES

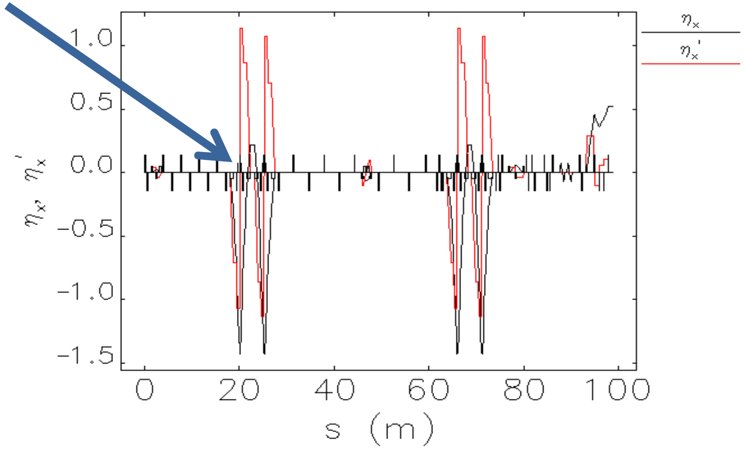


RIS



Local minimum of transverse aperture + dispersion





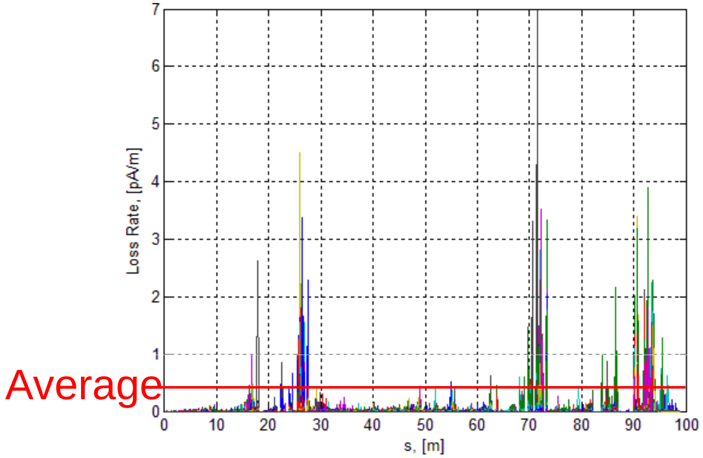
Residual Gas Scattering

Theoretical estimation

	RES(cERL)	RES(Cornell)	RIS (cERL)
theta_max	1.00E-03	1.00E-03	1.00E-2
Nbunch	4.80E+07	4.81E+07	4.80E+07
cross section	1.59E-24	2.92E-29	5.03E-29
velocity	3.00E+08	3.00E+08	3.00E+08
density	2.64E+14	2.64E+14	2.64E+14
Rate(Nparticle / s)	2.74E+06	4.84E+03	8.66E+01
Rate (pA/m)	4.38E-01	7.74E-04	1.39E-05

Blue=assumed

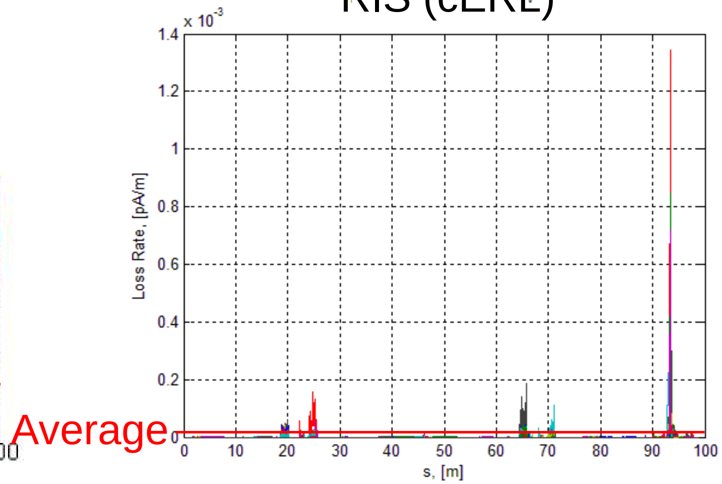
RES (cERL)



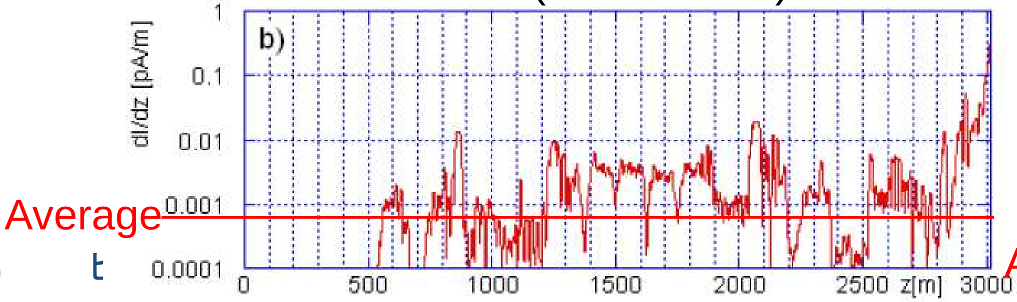
C. J. Bocchetta, CAS, 2003.

$$\sigma_{loss} = \frac{2\pi Z^2 r_p^2}{\gamma^2} \frac{1}{\theta_{\max}^2},$$

RIS (cERL)



RES(Cornell ERL)



A. B. Temnykh,EPAC,2008.

Beam loss studies at Compact ERL

Summary & Outlook

- We found the beam loss from all these effects in cERL are still not so significant, namely less the 1μA.
- Touschek effect should be taken into account mainly because is causes the radiation doses in the insertion devices.
- IBS would impact into beam losses for low energy operation mode or for very long beam transport.
- RGS includes to the beam losses, when pumping is not sufficient or situated at wrong places.

Field emission?!

Ion trapping?!

		TS	RES	RIS
cERL	Peak, pA/m	21	58	$1.3 \cdot 10^{-2}$
cERL	Average, pA/m	0.04	0.76	$5.9 \cdot 10^{-5}$
cERL theor.	Average, pA/m	0.11	0.44	$1.4 \cdot 10^{-5}$
Cornell***	Average, pA/m	0.6	$7 \cdot 10^{-4}$	
Cornell theor.	Average, pA/m	0.64	$7.74 \cdot 10^{-4}$	

ご清聴どうもありがとうございました

Appendices

Touschek Effect

Piwinski Loss Rate

Input parameters
rp=raduis of the particle=2.8179e-15 [m]
c=3e8 [m/sec]
βx,y=betatron function (from init.twi file)
Np=number of particles per bunch= =charge in bunch/electron charge
β=0.99967 (for Emax=20MeV)
γ=39.12894 (for Emax=20MeV)
me=0.511[MeV]
Dx,y=dispersion function (from init.twi file)
σxβ,σyβ=horizontal & vertical betatron width =√(βx,y*emitx,y)
emitx,y=normalized emittance=1e-6 [mm-mrad]
σs=rms bunch length=6e-4 [m]
σp=relative momentum spread=1e-3

$$R[particles/s]=\frac{r_p^2 c\beta_x\beta_y\sigma_h N_p^2}{8\sqrt{\pi}\beta^2\gamma^4\sigma_{x\beta}^2\sigma_{y\beta}^2\sigma_s\sigma_p}F(\tau_m,B_1,B_2)$$

Momentum Aperture:

$$\tau_m=\beta^2\delta_m^2=\beta^2\left(\frac{\Delta p_m}{p}\right)^2$$

$$\sigma_h=\frac{\sigma_{x\beta}\sigma_{y\beta}\sigma_p}{\sqrt{\tilde{\sigma}_x^2\sigma_{y\beta}^2+\tilde{\sigma}_y^2\sigma_{x\beta}^2-\sigma_{x\beta}^2\sigma_{y\beta}^2}}$$

$$F=\int_{\tau_m}^{\infty}\left(\left(2+\frac{1}{\tau}\right)^2\left(\frac{\tau/\tau_m}{1+\tau}-1\right)+1-\frac{\sqrt{1+\tau}}{\sqrt{\tau/\tau_m}}-\frac{1}{2\tau}\left(4+\frac{1}{\tau}\right)\ln\frac{\tau/\tau_m}{1+\tau}\right)e^{-B_1\tau}I_0(B_2\tau)\frac{\sqrt{\tau}\,d\tau}{\sqrt{1+\tau}}$$

$$B_1=\frac{\beta_x^2}{2\beta^2\gamma^2\sigma_{x\beta}^2}\left(1-\frac{\sigma_h^2\tilde{D}_x^2}{\sigma_{x\beta}^2}\right)+\frac{\beta_y^2}{2\beta^2\gamma^2\sigma_{y\beta}^2}\left(1-\frac{\sigma_h^2\tilde{D}_y^2}{\sigma_{y\beta}^2}\right)$$

$$\tilde{\sigma}_{x,y}^2=\sigma_{x\beta,y\beta}^2+\sigma_p^2\left(D_{x,y}^2+\tilde{D}_{x,y}^2\right)$$

$$\tilde{D}_{x,y}=\alpha_{x,y}D_{x,y}+\beta_{x,y}D'_{x,y}$$

$$B_2^2=B_1^2-\frac{\beta_x^2\beta_y^2\sigma_h^2}{\beta^4\gamma^4\sigma_{x\beta}^4\sigma_{y\beta}^4\sigma_p^2}\left(\sigma_x^2\sigma_y^2-\sigma_p^4D_x^2D_y^2\right)$$

IBS emittance growth rate simulation

Bjorken – Mtingwa formula

The growth times for the longitudinal phase space and momentum and bunc distributions are:

$$\frac{1}{\tau_p}=\frac{1}{2\sigma_p^2}\frac{d\sigma_p^2}{dt}=A\frac{\sigma_h^2}{\sigma_p^2}f(a,b,c)$$

$$\frac{1}{\tau_x}=\frac{1}{2\sigma_{x\beta}^2}\frac{d\sigma_{x\beta}^2}{dt}=A\frac{\sigma_h^2}{\sigma_p^2}\left[f\left(\frac{1}{a},\frac{b}{a},\frac{c}{a}\right)+\frac{\eta^2\sigma_p^2}{\sigma_{x\beta}}f(a,b,c)\right]$$

$$\frac{1}{\tau_y}=\frac{1}{2\sigma_{y\beta}^2}\frac{d\sigma_{y\beta}^2}{dt}=A\frac{\sigma_h^2}{\sigma_p^2}f\left(\frac{1}{b},\frac{a}{b},\frac{c}{b}\right)$$

with $A=\dfrac{r_e^2cN_b}{64\pi^2\sigma_s\sigma_p\sigma_{x\beta}\sigma_{y\beta}\sigma_{x'}\sigma_{y'}\beta^3\gamma^4}$ the particle bunch density.

Note γ^4 energy dependence

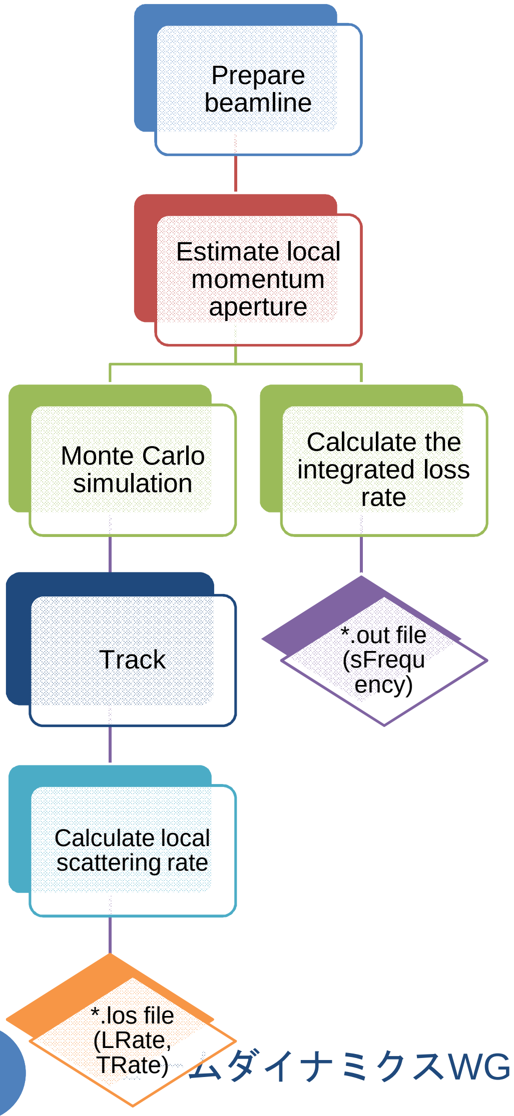
The function f is: $f(a,b,c)=8\pi\int_0^1\left\{\ln\left[\frac{c^2}{2}\left(\frac{1}{\sqrt{p}}+\frac{1}{\sqrt{q}}\right)\right]-0.577..\right\}\frac{1-3x^2}{\sqrt{pq}}dx$

and $p=\left(\frac{\sigma_h}{\gamma\sigma_{x'}}\right)^2+x^2\left(1-\left(\frac{\sigma_h}{\gamma\sigma_{x'}}\right)^2\right)$, $q=\left(\frac{\sigma_h}{\gamma\sigma_{y'}}\right)^2+x^2\left(1-\left(\frac{\sigma_h}{\gamma\sigma_{y'}}\right)^2\right)$

$$\sigma_h^2=\frac{\sigma_p^2\sigma_{x\beta}^2}{\sigma_{x\beta}^2+\alpha_c^2\sigma_p^2}\qquad c^2=\beta^2\sigma_h^2\frac{\sqrt{2\pi}\sigma_{y\beta}}{r_e}$$

Touschek scattering code analysis

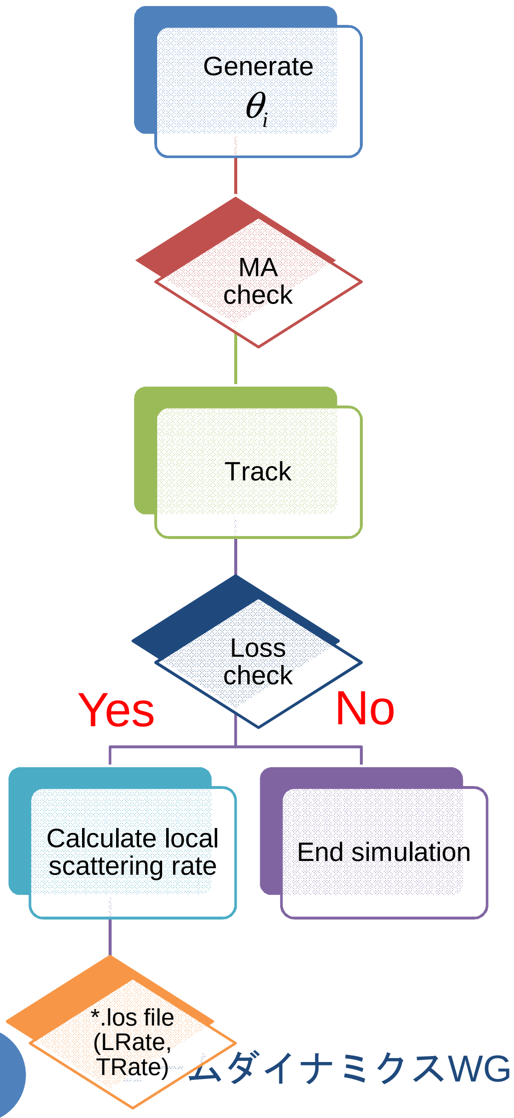
Routine structure

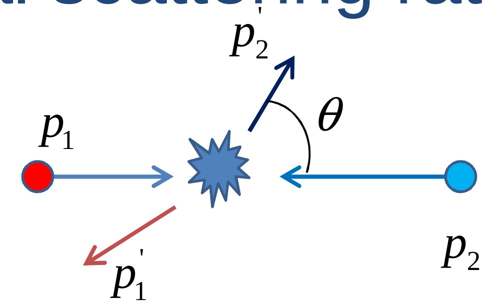


1. Prepare beamline: Insert scattering objects into beamline, upload beamline setup parameters
2. Estimate local momentum aperture: Twiss parameters and bunch parameters are required
3. Calculate the integrated loss rate from Piwinski's formula->sFrequency output
4. Monte Carlo Simulation: Generate scattered electrons using Monte Carlo method
5. Track simulated electrons to the end of the baemline
6. Calculate local scattering rate: Calculate the differential scattering rate from Piwinski's formula->LRate, TRate output
7. Record beam loss information

Touschek scattering code analysis

Monte Carlo simulation & differential scattering rate





1. θ_i is generated randomly for each electron track i ->$d\sigma(\theta_i)/d\theta$
2. Check the momentum aperture if $\frac{p_1' - p_0}{p_0} > \delta_m$ or $\frac{p_2' - p_0}{p_0} > \delta_m$ ->

   -> tracking; $p_0 = p\_central$ = reference momentum; $\left|\frac{\Delta p}{p}\right| = \delta_m$

3. Calculate differential scattering rate -> LRate, TRate output

$$w = \frac{dN}{dtd\Omega} = \frac{N_e^2 r_e^2 v_{rel}}{\sigma_x \sigma_y \sigma_s \gamma_{CM}^2} \frac{d\sigma}{d\Omega} = N_{beam} v_{rel} \rho_{target} \frac{d\sigma}{d\Omega}$$

β_{CM} – β of 2 particles within a bunch in CM sys.

γ_{CM} – γ of 2 particles within a bunch in CM sys.

Ne – number of electrons in bunch

$\sigma_x, \sigma_y, \sigma_s$ – bunch size

Ngen – number of generated particles

$d\sigma/d\Omega$ – differential Moeller cross-section

ρ_{target} - electron density in “target” bunch