TRANSVERSE RESISTIVE-WALL WAKE OF A ROUND PIPE WITH FINITE THICKNESS AND ITS EFFECT ON THE ERL MULTI-BUNCH BEAM

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Abstract

The expression of the exact transverse resistive-wall impedance of a round pipe with finite thickness was analytically derived and then the transverse resistive-wall wake functions of three pipes were numerically calculated from the impedance expression. Each of the calculated wake functions rapidly decreases around a time corresponding to the peak frequency of the impedance real part as compared with the conventional wake function so far used. As a result, the multi-bunch beam position displacement due to the resistive-wall wake in an ERL vacuum pipe is saturated around the time and does not increase unlimitedly with time. The saturated position displacement can be easily corrected with some correction scheme.

INTRODUCTION

Beam breakup (BBU) due to transverse resistive-wall wake in linear accelerators was first studied with an approximated analytical method in 2004[1]. This study suggested that the resistive-wall BBU could be very serious for high current ERLs, though not so for FELs. Thus we started to study the effect of resistive-wall wake on the multi-bunch beam in an ERL-based light source[2].

We simulated the ERL multi-bunch beam motion in the resistive-wall pipes with a developed simulation program and compared the simulation results with analytical results[2]. The simulation results showed that the beam position displacement due to the resistive-wall wake increased with time and its time dependence was in good agreement with that analytically obtained for some special cases. However the resistive-wall wake function so far used was valid only in a limited time range and improper to the RWBBU(resistive-wall BBU) study for a longer time period.

In this paper, we show an exact expression of the transverse resistive-wall impedance of a round pipe with finite thickness and the resistive-wall wake functions of several different pipes numerically calculated from the exact impedance expression. We also present a simulation result of the multi-bunch beam motion in a resistive-wall vacuum pipe to evaluate the effect of transverse resistive-wall wake in an ERL.

RESISTIVE-WALL IMPEDANCE

The exact expression of the transverse resistive-wall impedance of a round pipe with a radius b and a thickness d was analytically derived in order to obtain the exact transverse resistive-wall wake function. The pipe is made of a material with an electric conductivity σ, a permittivity ε_0 and a magnetic permeability μ_0. The permittivity and permeability are assumed to be equal to those of vacuum. The derived transverse impedance (per unit length) of the dipole mode (the lowest mode) is as follows:

$$Z_\perp(\omega) = \frac{-i}{\pi\varepsilon_0 b^3\left\{\lambda\alpha\left(1+\dfrac{2\omega^2}{\lambda^2 c^2}\right)-\dfrac{b\omega^2}{2c}\right\}} \tag{1}$$

$$\alpha = \frac{J_2(\lambda b)N_1(\lambda(b+d)) - J_1(\lambda(b+d))N_2(\lambda b)}{J_1(\lambda b)N_1(\lambda(b+d)) - J_1(\lambda(b+d))N_1(\lambda b)}$$

$$\lambda = \frac{i+\mathrm{sgn}(\omega)}{\delta} \quad \left(\delta = \sqrt{\frac{2}{\sigma\mu_0|\omega|}}\right)$$

Here c, i, ω, and δ are the velocity of light, the imaginary unit, the angular frequency and the skin depth, and J_1, J_2, N_1, N_2 are the first-order and second-order Bessel functions of the 1st and 2nd kinds respectively. The $sgn(\omega)$ means the sign of ω. If one consider the frequency range satisfying the following conditions:

$$|\lambda|b >> 1, \quad |\lambda|d >> 1, \quad |\lambda| >> b\omega^2/c^2$$

or

$$\frac{c}{2\pi}\sqrt{\frac{\sigma Z_0}{b^2}} >> f >> \frac{1}{2\pi\mu_0\sigma b^2}, \quad f >> \frac{1}{2\pi\mu_0\sigma d^2} \tag{2}$$

where f is the frequency ($f=\omega/2\pi$), Eq. (1) is approximated as

$$Z_\perp(\omega) \cong \frac{1}{\pi\varepsilon_0 b^3\lambda c} = \frac{Z_0\delta}{2\pi b^3}\left\{\mathrm{sgn}(\omega)-i\right\}. \tag{3}$$

Here Z_0 is the vacuum impedance.

Figure 1 shows the real parts of the resistive-wall impedances of SS(stainless steel, σ=1.4x10^6 Ω^{-1}m^{-1}) and Al(aluminium, σ=3.5x10^7 Ω^{-1}m^{-1}) pipes calculated from Eq. (1) and Eq. (3). In the frequency range where the conditions of (2) are satisfied, the impedance of Eq. (1) is well approximated by that of Eq. (3), which is inversely proportional to the cube of the pipe radius and proportional to the skin depth. At lower and higher frequencies, it has different behaviours from that of Eq.

(3). In particular, it should be noted that, as the frequency decreases, the real parts of the exact resistive-wall impedances go down to zero, while those of the approximated impedances continue to increase. In the lower frequency range, the exact impedance depends on the pipe thickness and its real part has a larger value for a thicker thickness. Its dependence becomes weak when the pipe thickness *d* exceeds the pipe radius *b*.

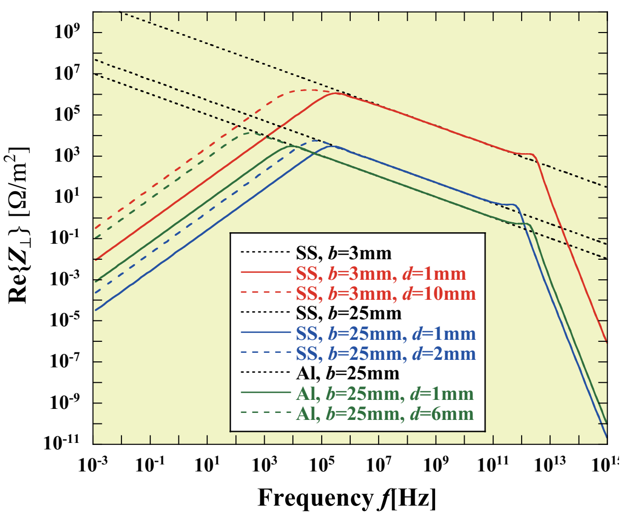

Figure 1: Real parts of transverse resistive-wall impedances of round SS and Al pipes with different radii and thicknesses (red, bule and green lines). The black dotted lines show real parts of the approximated impedances given by Eq. (3).

RESISTIVE-WALL WAKE FUNCTION

Wake function is generally expressed by using the impedance or its real part as follows:

$$\begin{aligned} W_\perp(z<0) &= \frac{-i}{2\pi}\int_{-\infty}^{\infty} Z_\perp(\omega)e^{i\omega z/c}d\omega \\ &= \frac{2}{\pi}\int_{-\infty}^{\infty} \mathrm{Re}\{Z_\perp(\omega)\}\sin(\omega z/c)d\omega \end{aligned} \tag{4}$$

Here z (< 0) is the longitudinal position from the wake source (an electron bunch) and $t=|z|/c$ is the elapsed time. The transverse wake function (per unit length) corresponding to Eq. (3) is given as function of the elapsed time t by

$$W_\perp(t) = -\frac{1}{\pi b^3 t^{1/2}}\sqrt{\frac{cZ_0}{\pi\sigma}} \tag{5}$$

By setting the frequency f to $1/t$, the conditions of (2) are rewritten to

$$\frac{2\pi}{c}\sqrt{\frac{b^2}{\sigma Z_0}} << t << 2\pi\mu_0\sigma b^2, \quad t << 2\pi\mu_0\sigma d^2 \tag{6}$$

Eq. (5) is a very famous expression and has been used for the RWBBU studies in Ref. [1] and [2]. However, the wake function of Eq. (5) is valid only in the time range where the conditions of (6) are satisfied.

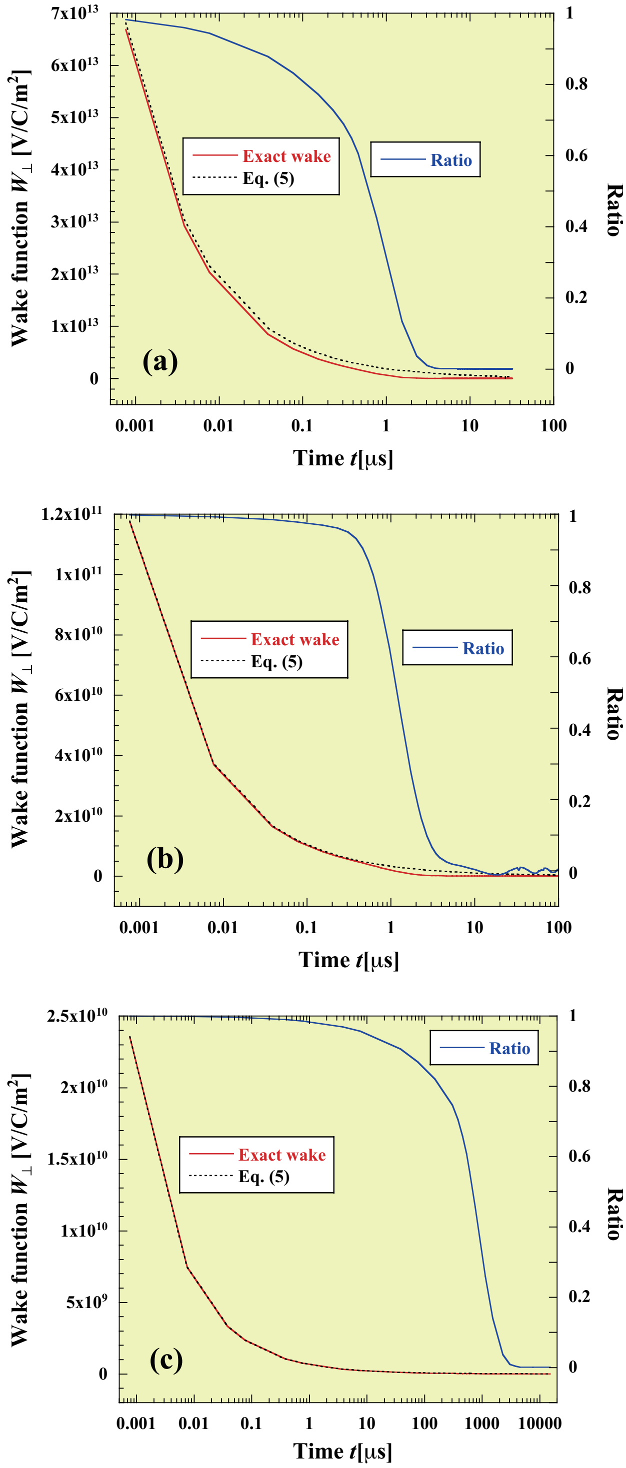

Figure 2: Transverse resistive-wall wake functions of (a) a SS pipe with b=3mm and d=1mm, (b) a SS pipe with b=25mm and d=1mm and (c) an Al pipe with b=25mm and d=6mm. The red solid lines and black dotted lines show the wake functions obtained from the exact impedance and Eq. (5) respectively. The blue solid lines show their ratios.

The lower limit of t on the conditions of (6) can be usually neglected when long-range wake causing multi-bunch BBU is considered. On the other hand, the upper limit of t imposes a severe condition on the equation of motion and the computer simulation using Eq. (5). The time ranges where Eq. (5) is valid are obtained for two round pipes (a SS pipe with b=3mm and d=1mm and an Al pipe with b=25mm and d=6mm) from conditions of (6) as follows:

$$5.4\times10^{-13} << t << 1.1\times10^{-5}\ (\mathrm{SS}, b=3\mathrm{mm}, d=1\mathrm{mm})$$
$$7.6\times10^{-13} << t << 1.1\times10^{-2}\ (\mathrm{Al}, b=25\mathrm{mm}, d=1\mathrm{mm})$$

Considering these small values of the upper time limit, exact wake functions should be calculated from the exact impedance expression of Eq. (1) in order to study the beam behaviour due to the resistive-wall wake for a longer time span.

We numerically calculated exact wake functions from Eqs. (1) and (4). Red solid lines in Figs. 2(a) to 2(c) show the calculated exact wake functions for three different pipes: SS pipe with b=3 mm and d=1 mm; SS pipe with b=25 mm and d=1 mm; Al pipe with b=25 mm and d=6 mm. A black dotted line and a blue solid line in each figure show Eq. (5) and the ratio of the exact wake function to Eq. (5) respectively. As clearly shown by the ratio of the wake function to Eq. (5), each exact wake function approximates Eq. (5) at small times where the conditions of (6) are satisfied and, compared with Eq. (5), drastically decreases around the time the reciprocal of which is equal to the peak frequency of the impedance real part in Fig. 1. After the drastic decrease, the wake function further decreases with time, oscillating with a period peculiar to each pipe, though it is not clearly seen here because of the large vertical scale of the figure.

RWBBU SIMULATION

The exact wake functions obtained in the previous section enable us to more accurately simulate transverse multi-bunch beam motion due to the resistive-wall wake in an ERL with the developed simulation program than Eq. (5). Here we consider a SS pipe with b=3mm and d=1mm, though the pipe radius is small for an ERL vacuum duct. The pipe length L is assumed here to be 56.44 m, which is equal to the return loop length of the compact ERL planned in Japan[3]. In the simulation, the multi-bunch beam with an energy of 60 MeV, a current of 100 mA and a bunch repetition rate of 1.3 GHz is injected into the pipe and all the bunches of the beam have the same transverse position offset y_0 at the pipe entrance. Figure 3 shows the simulated beam position $y(t)$ at the pipe exit normalized by the position offset y_0. The red solid line and the blue broken line indicate the normalized beam positions simulated by using the exact wake function in Fig. 2(a) and Eq. (5) respectively. Although the beam position displacement due to the exact wake function increases at small times, it is saturated a few microseconds after the beam injection start. Such a saturation of the beam position displacement is founded for the other pipes and is considered to be a common behaviour of the ERL multi-bunch beam moving in a resistive-wall pipe when the exact wake function is used. This means that, since the resistive-wall wake cumulated by passage of the multi-bunch beam is saturated, the beam position increase associated with RWBBU is limited within a finite range. On the other hand, the beam position displacement continues to increase unlimitedly with time if Eq. (5) is used as wake function.

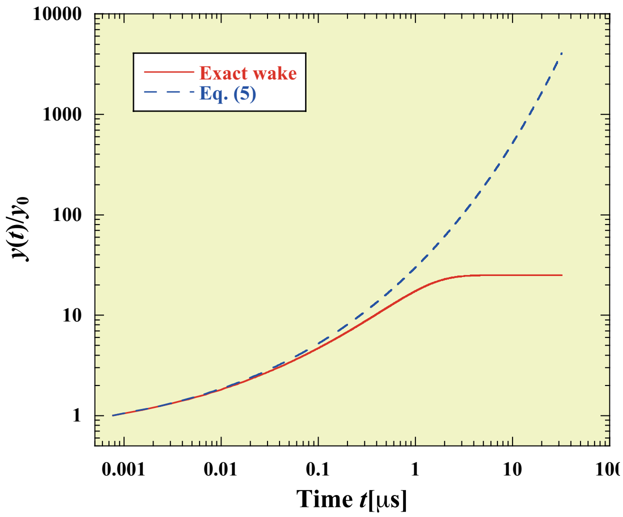

Figure 3: Simulated beam positions $y(t)$ at exit of a SS pipe with L=56.44m b=3mm and d=1mm normalized by the initial position offset y_0 as function of time. The red solid line and the blue broken line show the normalized positions for the calculated exact wake function in Fig. 2(a) and Eq. (5) respectively.

CONCLUSIONS

We succeeded in deriving analytical expression of the exact transverse resistive-wall impedance of a round pipe with finite thickness and obtaining the exact transverse resistive-wall wake functions of three different pipes. We also simulated the multi-bunch beam motion in a resistive pipe by use of the obtained exact wake functions. The simulated beam motion is saturated within a limited transverse position range because the cumulative effect of the resistive-wall wake becomes very weak with time. Therefore it is expected that the beam motion due to the resistive-wall wake in an ERL after the beam injection start can be well corrected with some correction scheme.

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