APPLICATION OF THE EIGENVECTOR METHOD WITH CONSTRAINTS TO ORBIT CORRECTION FOR ERLS

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Abstract

The eigenvector method with constraints(EVC method) is successfully applied to orbit correction for the compact ERL. Orbit distortion generated by the position error of eight main superconducting cavities is well corrected by the EVC method. Exact correction of local orbits can be achieved by the constraint conditions without using many eigenvectors that may cause local orbit distortion in the regions without any BPM and hence emittance growth. This method can be highly useful for orbit correction and stabilization in future ERL-based light sources.

INTRODUCTION

Orbit correction in an ERL is more complicated than those of an ordinary linac and a transport line, because the ERL beam passes a long section containing main superconducting cavities at least two times with different energies. A corrector in this section gives a different kick angle to the beam in a different turn. Therefore a sophisticated orbit correction method is required for ERLs The eigenvector method with constraints (hereafter called the EVC method)[1] can perform global orbit correction under constraint conditions and has been proposed for uniting global and exact local orbit corrections mainly in storage-ring based SR sources. In fact this method was successfully applied to orbit correction in the PF ring and PF-AR[2]. In this paper, we present the EVC method for ERLs and simulation results of orbit correction for the compact ERL.

EIGENVECTOR METHOD WITH CONSTRAINTS

Principle

When the vector of a distorted orbit $\vec{x}$ at beam position monitors(BPMs) is corrected by corrector kick angles $\vec{\theta}$, the residual orbit $\vec{\Delta}$ is expressed as

$$\vec{\Delta} = R\vec{\theta} + \vec{x}\,. \tag{1}$$

Here the dimensions of the vectors $\vec{x}$ and $\vec{\theta}$ are M and N and R is the $M \times N$ response matrix. Now constraint conditions are given by

$$\overrightarrow{C_i}^T \cdot \vec{\theta} + z_i = 0 \;(\, i = 1, 2\cdots, N_c \,)\,, \tag{2}$$

where the superscript T stands for the transposed matrix or vector and z_i is a parameter related to the kick angles. N_c and $\vec{C_i}^T$ are the number of the constraints and the response matrix between $\vec{\theta}$ and z_i.

The norm of $\vec{\Delta}$ should be minimized or sufficiently small under the constraint conditions of Eq. (2) by introducing the following function of S (Lagrange's method of indeterminate multipliers).

$$S = \frac{1}{2}\left(R\vec{\theta} + \vec{x}\right)^2 + \sum_{i=1}^{N_c} \mu_i \left(\overrightarrow{C_i}^T \cdot \vec{\theta} + z_i\right) \tag{3}$$

Derivatives of the function S with respect to all the elements of $\vec{\theta}$ and $\vec{\mu}$ are set to zero and the following equations are obtained.

$$A\vec{\theta} + R^T \vec{x} + C^T \vec{\mu} = 0 \tag{4}$$

$$C\vec{\theta} + \vec{z} = 0 \tag{5}$$

where

$$A = R^T R \tag{6}$$

$$\vec{z} = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_{N_c} \end{pmatrix}, \quad \vec{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_{N_c} \end{pmatrix} \tag{7}$$

$$C = \begin{pmatrix} \overrightarrow{C_1}^T \\ \overrightarrow{C_2}^T \\ \vdots \\ \overrightarrow{C_{N_c}}^T \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \vdots & \vdots & & \vdots \\ C_{N_c 1} & C_{N_c 2} & \cdots & C_{N_c N} \end{pmatrix} \tag{8}$$

Eq. (5) is identical with the constraint conditions of Eq. (2). The solutions of $\vec{\mu}$ and $\vec{\theta}$ are derived from Eqs. (4) and (5).

$$\vec{\mu} = P^{-1}\vec{z} - P^{-1}CA^{-1}R^T\vec{x} \tag{9}$$

$$\vec{\theta} = B\vec{x} - D\vec{z} \tag{10}$$

where

$$B = (-A^{-1} + A^{-1}C^T P^{-1} C A^{-1})R^T \tag{11}$$

$$D = A^{-1}C^T P^{-1} \tag{12}$$

$$P = CA^{-1}C^T \tag{13}$$

The matrix A^{-1} is a generalized inverse matrix defined by

$$A^{-1} = \sum_{i=1}^{N_v} \frac{\vec{v_i}\,\vec{v_i}^T}{\lambda_i} \quad \left(N_v \le N\right). \tag{14}$$

Here λ_i (≥0) and $\vec{v}_i$ are the i-th largest eigenvalue and its eigenvector of the matrix A. For $\lambda_i \approx 0$, $1/\lambda_i$ in Eq. (14) is often replaced with zero to avoid very large kick angles and N_v is usually unequal to N. The condition of $N_v \ge N_c$ is required for the existence of the inverse matrix P^{-1}. More details of the EVC method are described in ref. [1].

If $\vec{z}$ is taken as the electron (or photon) beam positions measured at arbitrarily selected BPMs (or photon BPMs) and C as the corresponding response matrix, the beam positions at the selected BPMs are fixed at zero by this corection. For the electron beam and BPMs, Eq. (10) can be rewritten in a simplified form.

$$\vec{\theta} = G\vec{x}\ , \tag{15}$$

where G is an N x M matrix.

Response Matrix for ERLs

The EVC method can be applied to ERLs only by replacing the response matrix. Here, for a 1-loop ERL, number of beam positions monitored at all the BPMs is M and number of all the correctors are N. The betatron functions and phases at the i-th position monitored by its corresponding BPM and the j-th corrector are (β_i, ϕ_i) and (β_j, ϕ_j), respectively. The response matrix element R_{ij} between the i-th position and the kick angle of the j-th corrector is usually expressed by

$$R_{ij} = \sqrt{\frac{p_j}{p_i}}\sqrt{\beta_i\beta_j}\ sin\left(\phi_i - \phi_j\right), \quad \phi_i > \phi_j \tag{16}$$

where p_i and p_j are the beam momenta at the i-th position and the j-th corrector, respectively. In the 2nd turn, the j-th corrector that the beam passes two times provides the (j+L)-th kick as well as the j-th kick. Here L is the total number of the correctors in the ERL loop (not including the injector and extractor sections). In this case, the response matrix element R_{ij} is given by

$$\begin{aligned} R_{ij} = &\sqrt{\frac{p_j}{p_i}}\sqrt{\beta_i\beta_j}\ sin\left(\phi_i - \phi_j\right) \\ &+ \sqrt{\frac{p_{j+L}}{p_i}}\sqrt{\beta_i\beta_{j+L}}\ sin\left(\phi_i - \phi_{j+L}\right), \\ &\qquad \phi_i \geq \phi_{j+L} > \phi_j \end{aligned} \tag{17}$$

Here p_{j+L}, β_{j+L} and ϕ_{j+L} are the beam momentum and the betatron function and phase at the j-th corrector in the second turn. The magnetic field integral of the corrector is often a better parameter than the kick angle because it does not depend on the beam momentum. When the magnetic field integral is used in place of the kick angle, the elements of the response matrix corresponding to Eqs. (16) and (17) are given by

$$R_{ij} = e\sqrt{\frac{\beta_i\beta_j}{p_i p_j}}\ sin\left(\phi_i - \phi_j\right), \quad \phi_i > \phi_j \tag{18}$$

and

$$\begin{aligned} R_{ij} = &e\sqrt{\frac{\beta_i\beta_j}{p_i p_j}}\ sin\left(\phi_i - \phi_j\right) \\ &+ e\sqrt{\frac{\beta_i\beta_{j+L}}{p_i p_{j+L}}}\ sin\left(\phi_i - \phi_{j+L}\right), \quad \phi_i \geq \phi_{j+L} > \phi_j \end{aligned} \tag{19}$$

where e is the electron charge.

APPLICATION TO THE COMPACT ERL

The EVC method is applied to orbit correction in the compact ERL here. Layout of the compact ERL[3] including BPMs(BPM01-BPM23) and correctors(COR01 -COR19) is shown in Fig. 1. Twenty-three BPMs provides 28 beam positions(M=28) because five(BPM01-BPM05) of them between the merger and the extractor monitor two beam positions in the 1st and 2nd turns. Similarly five(COR01-COR05) of the nineteen correctors(N=19) give two kicks to the beam. The beam orbits are simulated by elegant[4] for the magnetic field integrals of the correctors obtained by the EVC program. The orbit distortion to be corrected by the EVC method is generated by position error of 1 mm for the eight main SC cavities. We choose constraint conditions that the two positions at BPM13 and BPM14 are zero, because the beam in the long straight section between the two BPMs should be particularly corrected and stabilized for user experiments.

Figure 2 shows a horizontal orbit distorted by the horizontal cavity position error of 1 mm in LE mode. The maximum positions are 34.8 and 26.6 mm and the RMS(root mean square) positions 8.43 and 7.85 mm for all the elements and BPMs, respectively. The orbit is corrected by the EVC method with the eigenvector number N_v=3 to 19. Figure 3a and 3b show dependence of RMS and maximum positions for all the elements and BPMs on the eigenvector number N_v. The RMS and maximum positions for all the BPMs become small as the eigenvector number goes up. On the other hand, the RMS and maximum positions for all the elements stop decreasing at N_v=6 or 7 and start increasing at N_v=8. This reason is explained by using Fig. 4. When the eigenvector number changes from 7 to 8, the orbit shape is significantly changed around the cavities as shown in Fig. 4. Although this change further reduces the orbit distortion at the BPMs, it increases the orbit distortion around the cavities where there is no BPM. This orbit shape change also degrades the emittance as shown in Fig. 5. The emittance is substantially increased at the cavities by the chromatic effects for the corrected orbit with $N_v \geq 8$, while being increased at the quadrupole magnets for uncorrected orbit[5]. In that point, the orbit correction with $N_v \leq 7$ is better than that with $N_v \geq 8$. In addition, the RMS and maximum field integrals of the correctors are smaller than those with $N_v \geq 8$ as shown in Fig. 6.

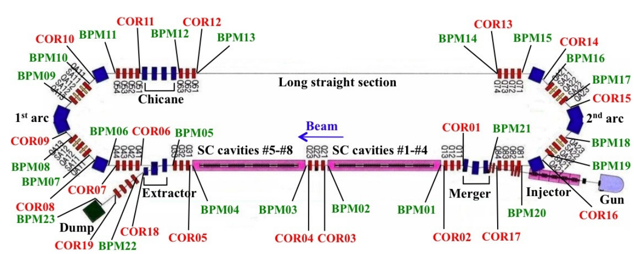

Figure 1: Layout of a 1-loop compact ERL.

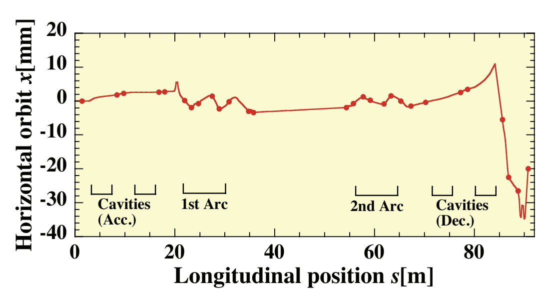

Figure 2: Horizontal orbit distorted by the cavity alignment error of 1 mm in LE mode. The solid circles indicate 28 beam positions at the BPMs.

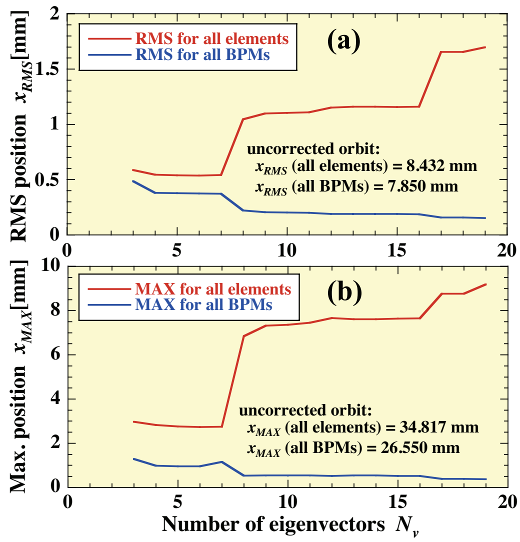

Figure 3: Dependence of (a) RMS and (b) maximum horizontal positions for all the elements and BPMs on number of eigenvectors used for orbit correction by the EVC method.

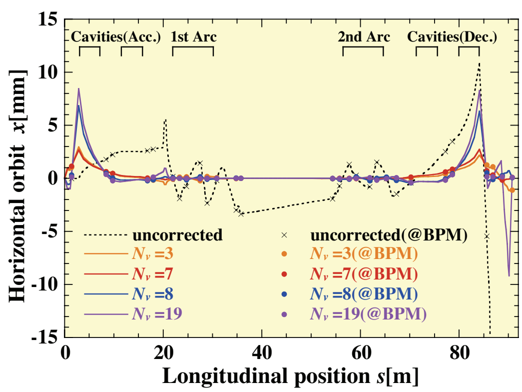

Figure 4: Horizontal orbits uncorrected and corrected by the EVC method with the eigenvector number N_v=3, 7, 8 and 19. The solid circles and crosses indicate the positions at all the BPMs.

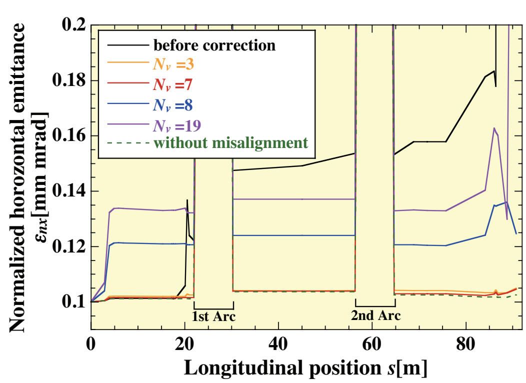

Figure 5: Horizontal normalized emittances in LE mode for orbits uncorrected and corrected by the EVC method with the eigenvector number N_v=3, 7, 8 and 19.

The RMS beam positions of the EVC method for all the elements and BPMs are equal within 5 % to those of the ordinary eigenvector method without constraints for $N_v \geq 3$. On the other hand, the beam positions at the two selected BPMs, BPM13 and BPM14, are much better corrected by using the constraint conditions as shown in Fig. 7. This is a great advantage of the EVC method. Exact orbit correction at locations where the orbit accuracy or stability is especially required can be achieved by the EVC method even if a large number of eigenvectors, which may cause local orbit distortion and/or emittance increase, is not used.

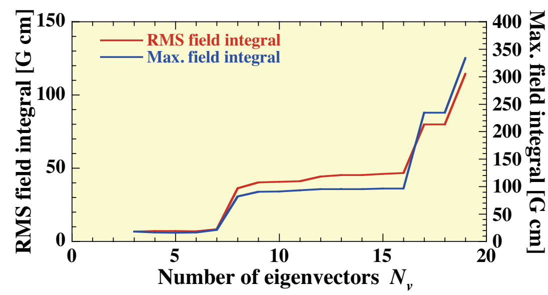

Figure 6: Dependence of RMS and maximum field integral of the correctors on number of eigenvectors.

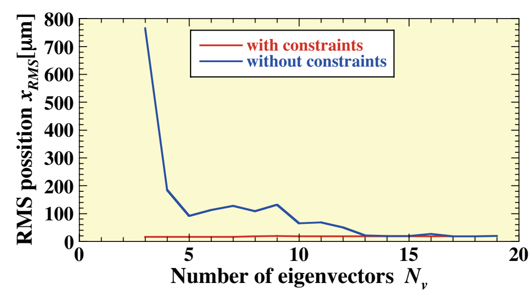

Figure 6: RMS beam positions for the two BPMs (BPM13 and BPM14) after orbit correction by the eigenvector method with and without constraints.

SUMMARY AND CONCLUSIONS

The eigenvector method with constraints successfully works for orbit correction of the compact ERL. The orbit distortion due to the cavity position error is globally reduced and exact correction of the local orbits is well achieved by the constraint conditions. This method can flexibly control orbits by using constraint conditions and adjusting the eigenvector number and it is suitable for orbit correction and stabilization in future ERL-based light sources that have many light source points.

REFERENCES

[1] N. Nakamura et al., Nucl. Instr. Meth. A **556** (2006) 421- 432.

[2] K. Harada et al., Nucl. Instr. Meth. A **604** (2009) 481-488.

[3] T. Shiraga et al., PAC09, Vancouver, 2009.

[4] M. Borland, Phys. Rev. ST-AB **4**, 070701 (2001).

[5] N. Nakamura et al., in these proceedings.