# Width of Interference Fringe in Bragg-(Bragg) ${ }^{m}$-Laue Case 

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The interference fringe in the Bragg-(Bragg) ${ }^{m}$-Laue ( $\mathrm{BB}^{\mathrm{m}} \mathrm{L}$ hereafter) case have been observed by Fukamachi et al.[1,2] from a thin absorbing crystal, when the effective linear absorption coefficient is zero due to the dynamical diffraction effect. In this paper, we report on the change in the period of the interference fringe.

Figure 1 shows the geometry of the $\mathrm{BB}^{\mathrm{m}} \mathrm{L}$ case. Some parts of the incident X-rays are confined and propagate in the crystal with repeated reflections. When the X-rays reach at the edge surface $B$, they come out in the reflection and transmission directions. Here $m$ is the number of reflections, $H$ crystal thickness and $L$ the distance from the position of the incident point O to the edge $B$. The interference fringes in the emitted beam from the surface $B$ were observed experimentally.

According to the dynamical theory of diffraction in the two-wave approximation, the X-rays excited at the dispersion point corresponding to the ingoing wave field (denoted as 1) can be confined in the $\mathrm{BB}^{\mathrm{m}} \mathrm{L}$ case. The Poynting vector $\boldsymbol{S}^{(1)}=\left(\boldsymbol{s}_{0}^{(1)}+\boldsymbol{s}_{h}^{(1)}\right) \boldsymbol{D}_{0}^{(1)} \boldsymbol{D}_{h}^{(1)}$ propagates from the incident point O to the exit point Q through the path OQ. Here the superscript (1) represents the dispersion point $1, \boldsymbol{s}_{0}^{(1)}$ and $\boldsymbol{s}_{h}^{(1)}$ are the unit vectors in the reflection and transmission directions, and $\boldsymbol{D}_{0}^{(1)}$ and $\boldsymbol{D}_{h}^{(1)}$ the electric displacements in the crystal corresponding to the transmitted and reflected beams, respectively. Suppose that the Poynting vector $\boldsymbol{S}^{(2)}$ excited at the dispersion point 2 corresponding to the outgoing wave filed propagates from the point $\mathrm{O}^{\prime}$ to the point Q through the path O'Q, the electric components of $\boldsymbol{S}^{(1)}$ and $\boldsymbol{S}^{(2)}$ overlap at the point Q . Then the width of the interference fringe $\Lambda_{B}$ at the point $Q$ on the surface $B$ can be determined by,

$$
\begin{equation*}
\Lambda_{B}=2 \Lambda \frac{L}{H} \tan ^{2} \theta_{B} \tag{1}
\end{equation*}
$$

where $\theta_{B}$ is the Bragg angle and $L \gg H$ is assumed. $\Lambda$ is the width of Pendellösung fringe and is given by

$$
\begin{equation*}
\Lambda=\frac{1}{\kappa_{0} \cos \theta_{B}\left(\chi_{h} \chi_{-h}\right)^{1 / 2}} \tag{2}
\end{equation*}
$$

Here, $\quad \chi_{h}$ is the Fourier component of X-ray polarizability and $\kappa_{0}$ the average wave number in the crystal. In figure 2, the measured [2] and calculated fringe widths $\Lambda_{B}$ are shown as a function of $L$. The measured and calculated values show good agreement; the values
are approximately the same and increase linearly as a function of $L$. The measured values of $\Lambda_{B}$ is inversely proportional to $H$ and $\left|\chi_{h}\right|$. It is clear that eq. (1) explains the measured variation at least qualitatively.

The present results should be useful for the analysis of diffraction scheme of the $\mathrm{BB}^{\mathrm{m}} \mathrm{L}$ case.

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Fig. 1. Illustration of the geometry of ${B B^{m}}^{L}$ case.


Fig. 2. Relation between the period of interference fringe $\Lambda_{B}$ and the distance $L$. Squares are the observed values and the solid line is the calculated values using (1) for Ge 222 reflection. The crystal thickness is $38 \mu \mathrm{~m}$.

## References

[1] T. Fukamachi et al., JJAP 43,L865(2004).
[2] T. Fukamachi et al., JJAP 44,L787(2005).

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