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Statistics of Measured Intensities Affected by Counting Loss in Detection Systems

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Introduction

Counting method is widely used in evaluation of X-ray beam intensities. The measured counts are always affected by counting loss, which may be caused by finite response time of the detector and electronic circuits in detection systems. In our previous report [1], we have presented practical methods for experimental characterization and correction of the systematic deviation caused by the counting loss expressed by non-extended and extended dead-time models, and also an intermediately extended dead-time model between the two conventional models. Recently, Laundy & Collins [2] have proposed a pulse-overlap model for counting loss, including the extended dead-time model as a special case. In this study, the statistical properties of the non-extended and extended dead-time models are examined, applying analytical formulas and Monte Carlo simulation.

Statistics for Counting Loss

The probability that n counts are detected for the pulses that obeys the Poisson distribution with average rate r for the measurement time T on the non-extended dead-time model with dead-time τ is given by

$$P_{\mathrm{non}}(n)=(n!)^{-1}(1+r\tau)[rT(1-n\tau/T)]^{n}\exp[-rT(1-n\tau/T)]$$

The mean and variance of the above formula are given by

$$\mu_{\mathrm{non}} = rT\,/\,(1+r\tau)$$
$$\sigma_{\mathrm{non}}^{2} = \mu_{\mathrm{non}}\,/\,(1+r\tau)^{2}$$

The mean and variance of the counted pulses affected by the extended dead-time model are given by [2]

$$\mu_{\mathrm{ext}} = rT\exp(-r\tau)$$
$$\sigma_{\mathrm{ext}}^{2} = \mu_{\mathrm{ext}}\,[1-2r\tau\exp(-r\tau)]$$

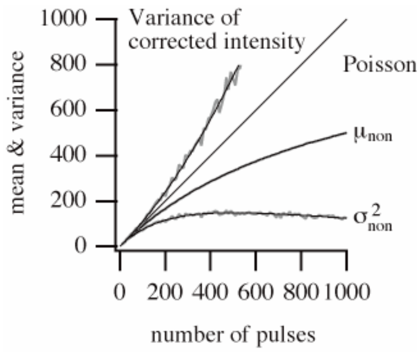
Note that the variance of the counted pulses on both non-extended and extended dead-time models are lower than the mean, while the variance should be equal to the mean in case of Poisson distribution.

Simulation

To simulate the pulses that obeys the Poisson distribution, a pulse is generated at the time $t = t_0+\Delta t$ after random period Δt from the time t_0 when the last pulse was generated, where Δt is calculated by $\Delta t=-\ln(x)/r$ from a random number x evenly distributed between 0 and 1.

The pulse is counted when $t - t_0 > \tau$ on the extended model, and $t - t_v > \tau$, where t_v is the arrival time of the last counted pulse, on the non-extended model. The process is repeated while $t < T$.

The results of simulation and theoretical prediction are shown in Fig. 1. The mean and variance were calculated from 1000 trials of simulation for each rate of pulses. The statistical errors of the corrected intensity rapidly increase on increasing rate of pulses, but they can reasonably be estimated by the theoretical models.



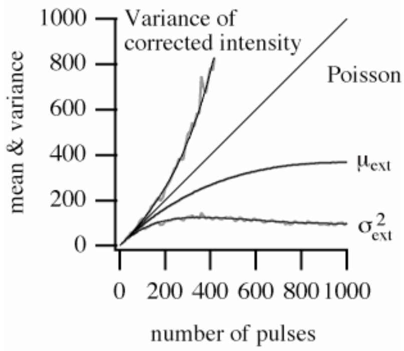


Fig. 1: Mean and variance of the observed and corrected counts for r=1000, T=1, τ=0.001 on the non-extended (upper) and extended (lower) models. The results of simulation and theoretical prediction are respectively drawn as thick gray and thin black lines.

References

[1] T. Ida et al., PF Activity Report 2005 Part B, 252 (2006).
[2] D. Laundy et al., J. Synchrotron Rad. 10, 214 (2003).

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