Development of Optical Systems for Hard X-rays at SPring-8

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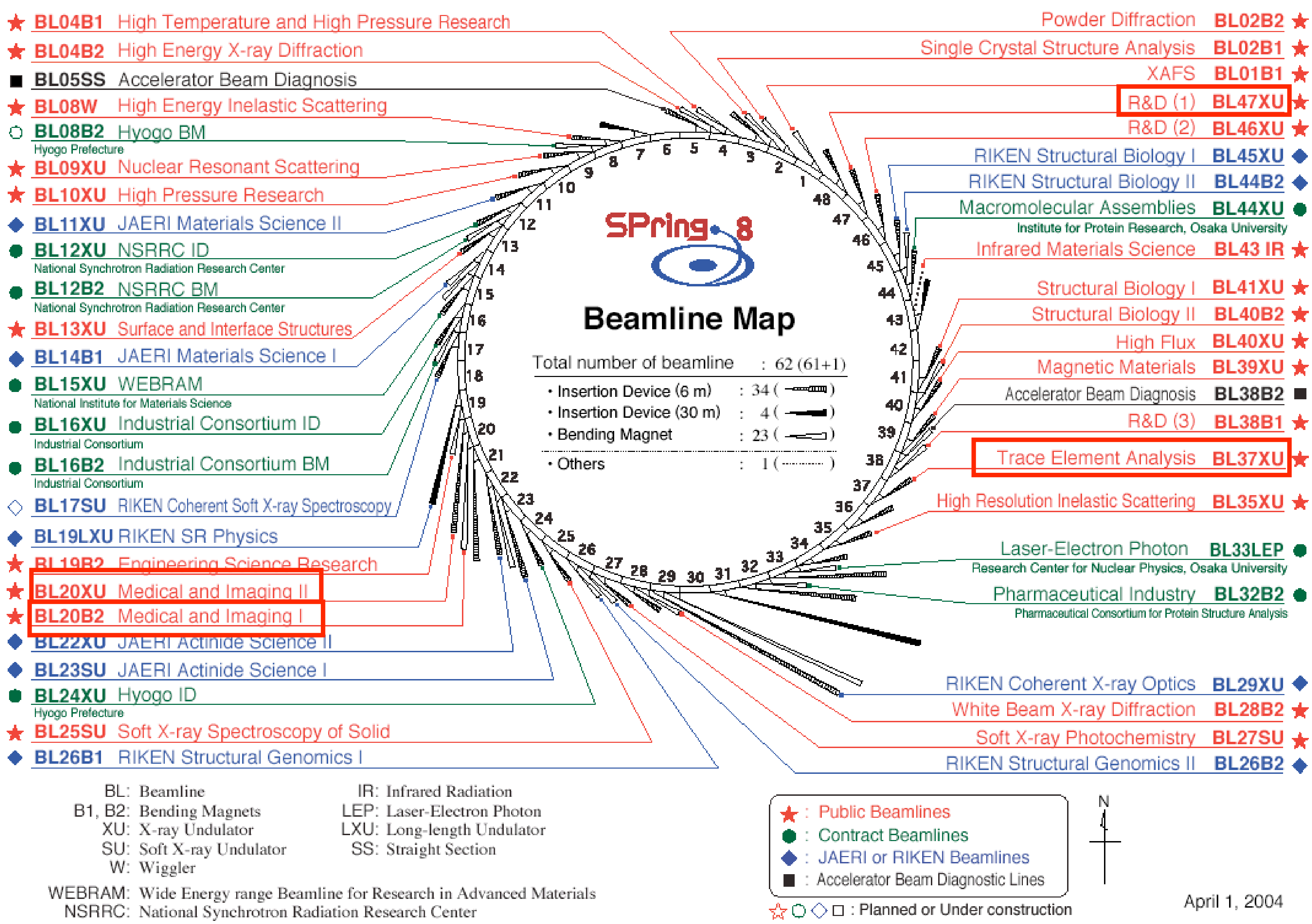
Development of Hard X-ray Optics at SPring-8

1. Beamline configurations,
2. Fresnel zone plate optics
   Micro- & nano-beam generation and characterization,
   Imaging microscopy and tomography applications,
   Holography and phase-contrast imaging,
   Use of quasi-monochromatic (direct undulator) radiation,
3. Total reflection mirror optics,
4. Sputtered-sliced zone plate optics,
5. Refractive lens optics,
6. Theory of resolution limit.



BL: Beamline
B1, B2: Bending Magnets
XU: X-ray Undulator
SU: Soft X-ray Undulator
W: Wiggler
IR: Infrared Radiation
LEP: Laser-Electron Photon
LXU: Long-length Undulator
SS: Straight Section

WEBRAM: Wide Energy range Beamline for Research in Advanced Materials
NSRRC: National Synchrotron Radiation Research Center

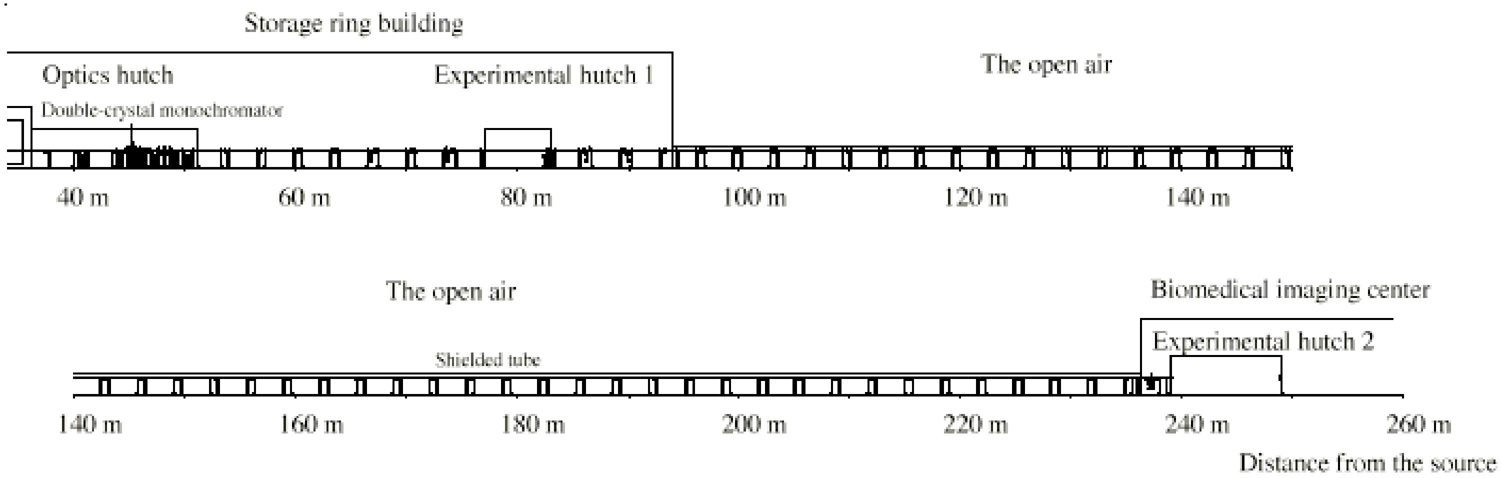
April 1, 2004

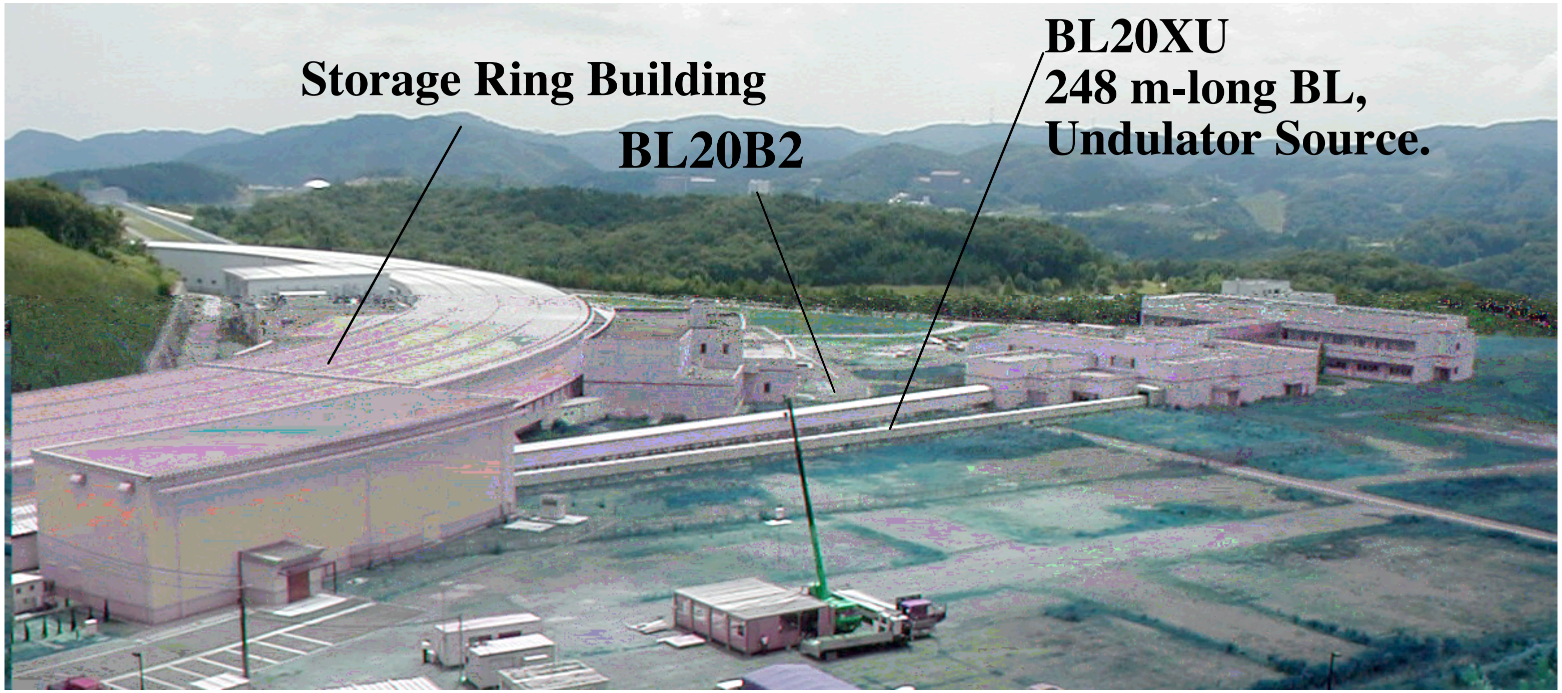
Overview of Beamlines

BL20XU (Coherent optics, Microbeam, Holography, Interferometer, etc.)
Undulator, Medium Length (248 m) beamline.
BL20B2 (Computer Tomography, Topography, Medical Imaging, etc.)
Bending Magnet, Medium Length (215 m).
BL37XU (Fluorescent X-ray Micro-analysis)
Undulator, Normal Length (50 m) beamline.
BL47XU (Microbeam, Imaging Microscopy & Micro-CT)
Undulator, Normal Length (50 m) beamline.

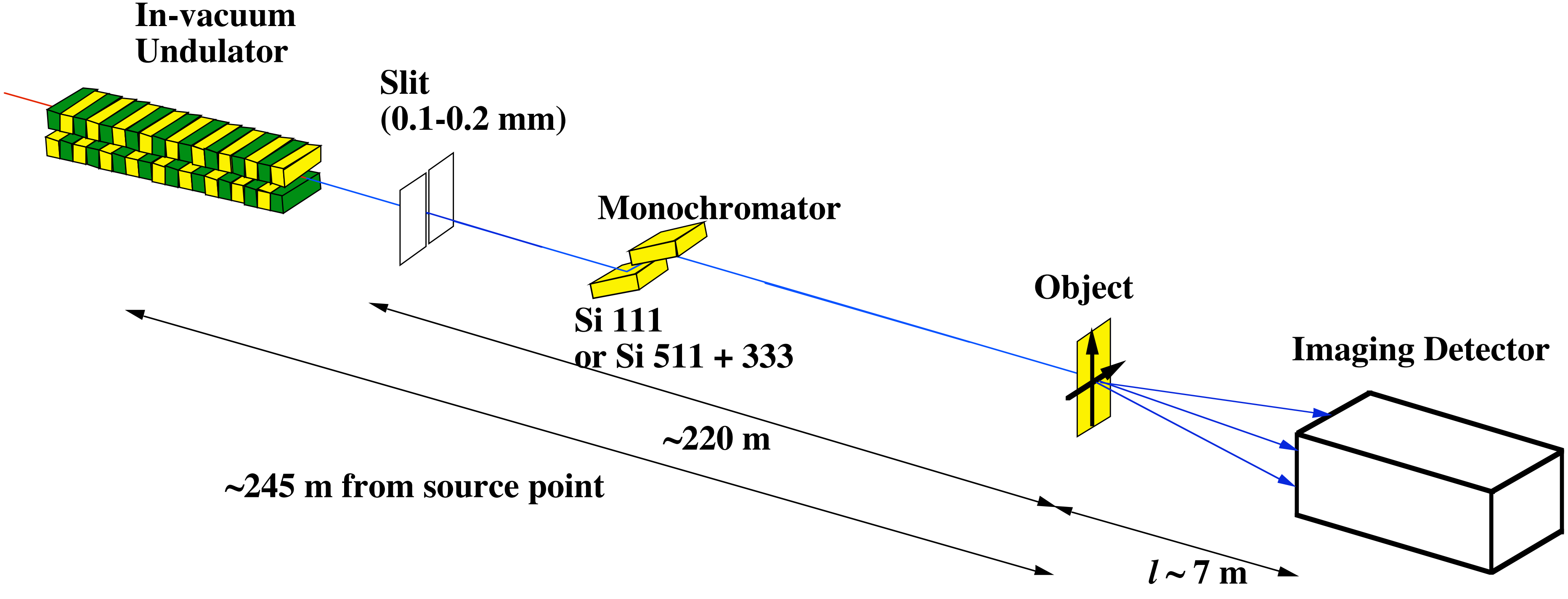
(BL28B2: BM white beam, topography, BL29XU: RIKEN, 1 km BL)

Energy range:
5-113 keV,
Spatial Coherence:
~ 1 mm @ 1Å (BL20XU),
Beam Cross-section:
~ 30 nm (micro-focus) - 300 mm (215 m-beamline with BM source).
Flux Density:
~ 10E14 photons/s/mm2 (direct beam),
> 10E11 photons/s/μm2 (micro-focus).



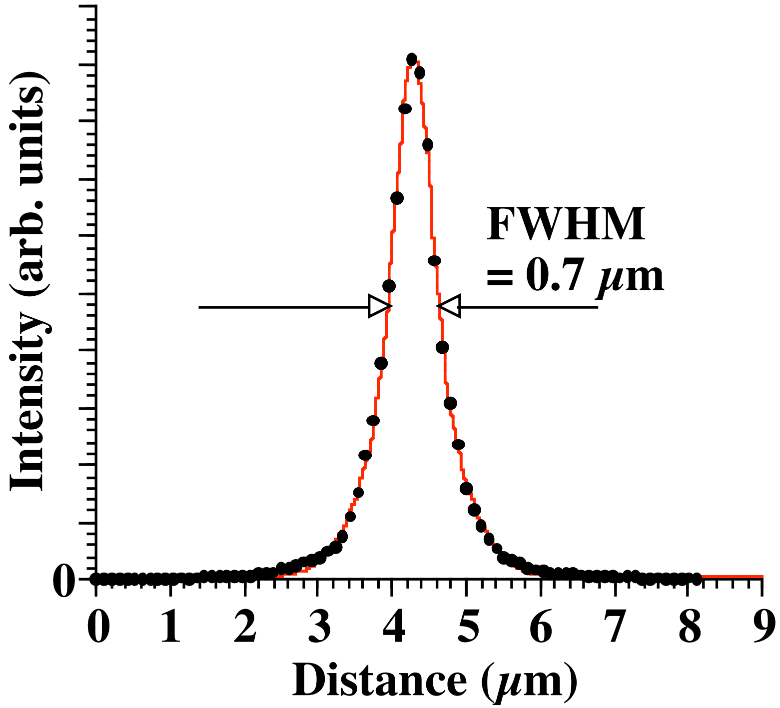


248 m-long Beamline, BL20XU, at SPring-8



Experimental setup

Application of Coherent X-ray Beam
-X-ray In-line Holography -



Measured Point-spread-function

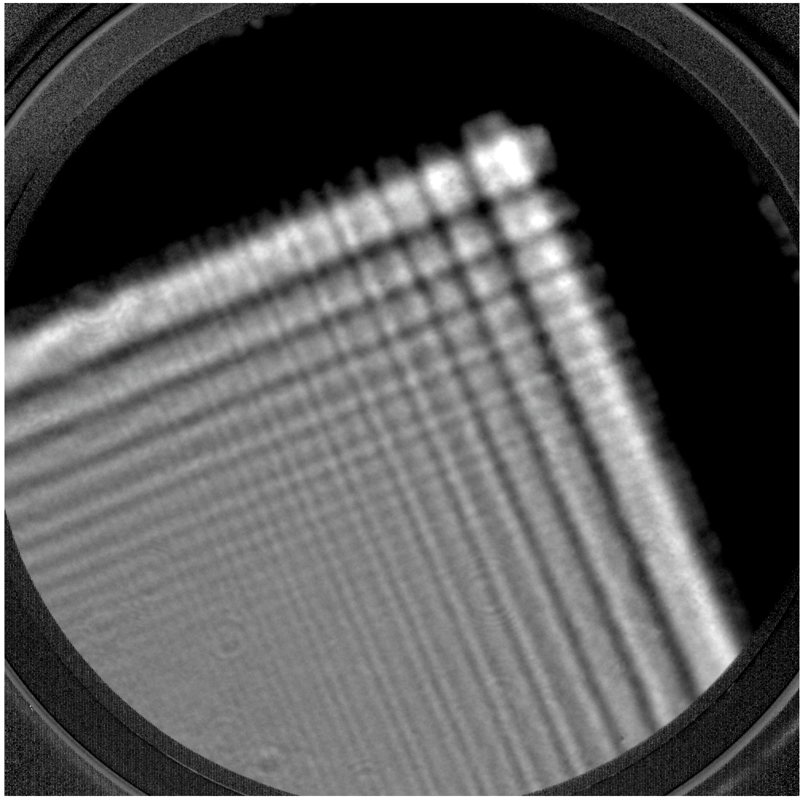
"Zooming Tube"

Hamamatsu Photonics C5333

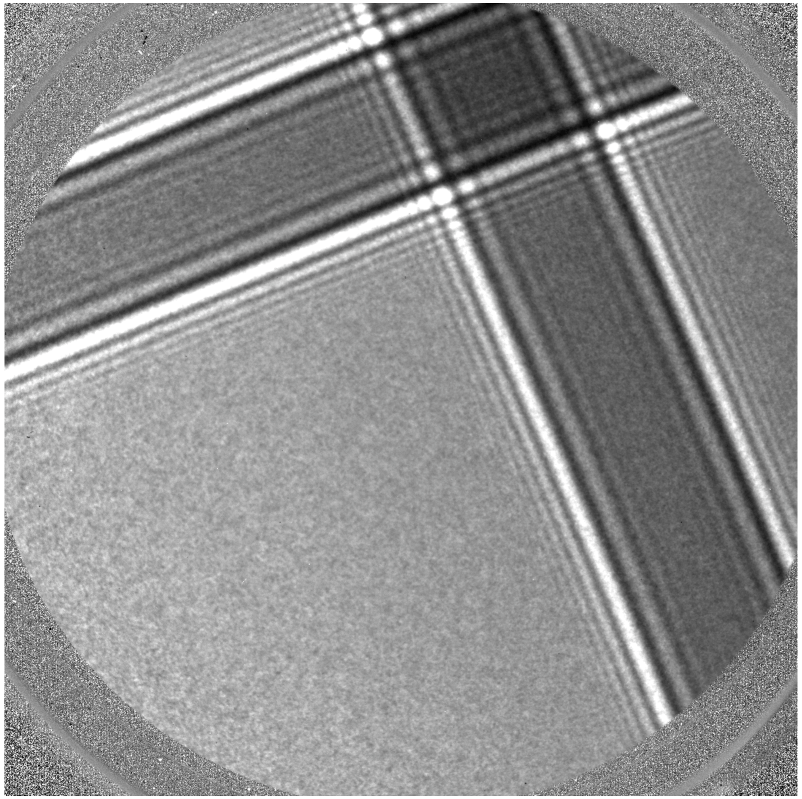
Photo-cathode: CsI (~2000Å),

Magnification: 5-240,

Resolution (point-spread-function): 0.7 μm in FWHM.



EX-ray Energy: 8 keV

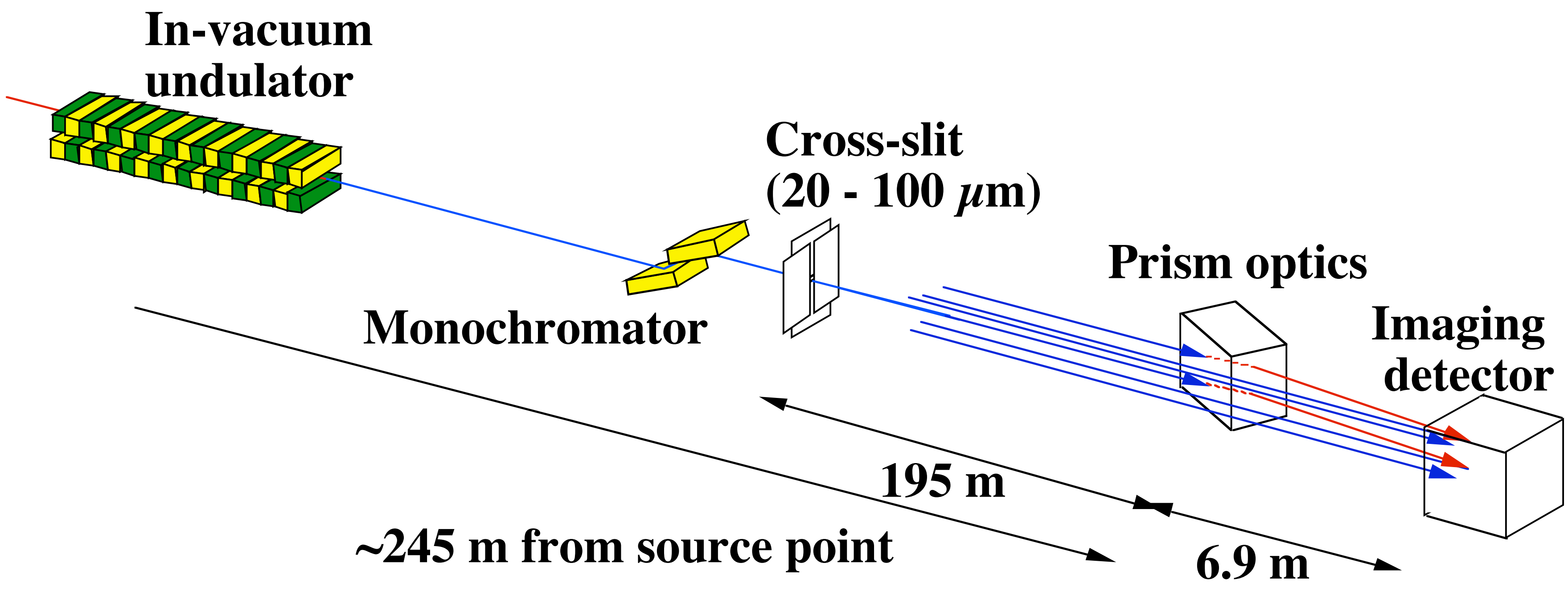


X-ray Energy: 80 keV

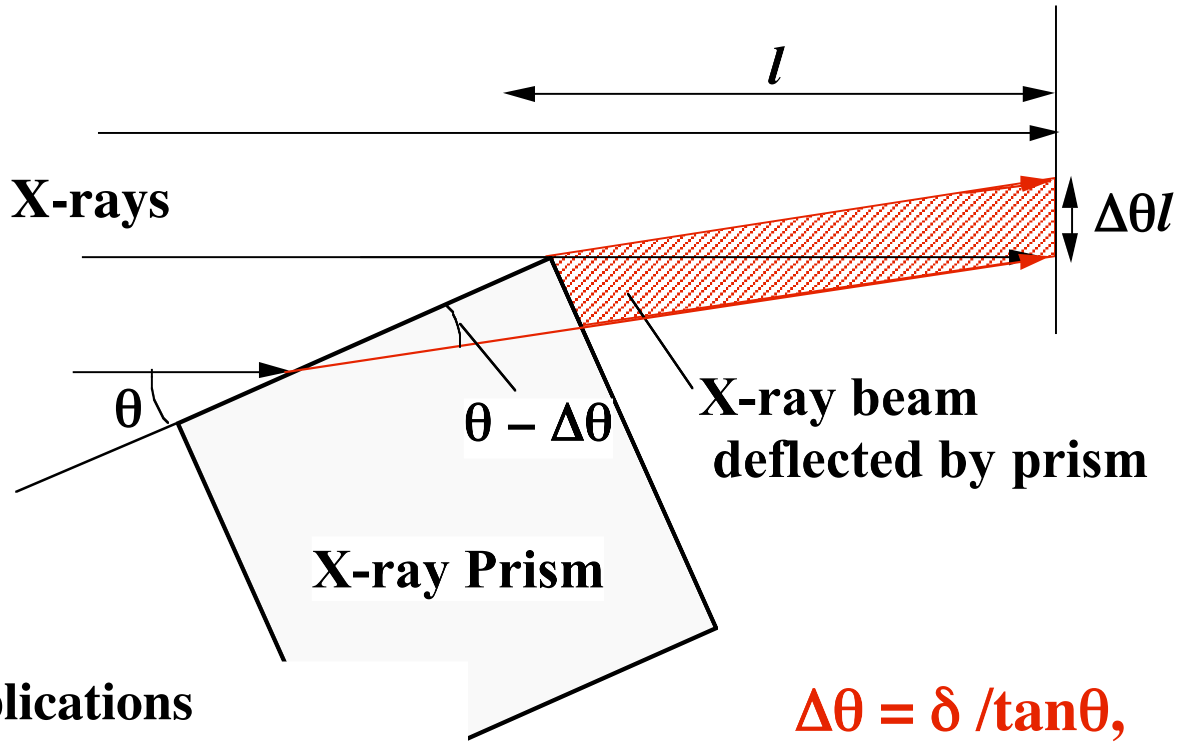
Object: Gold Wire, 50 μm Diameter,
Imaging Detector: Zooming Tube, C5333, Hamamatsu Photonics,
Field of View: 300 μm in Diameter.

Measured Hologram of Test Object

Measurement of X-ray Coherence using Two-beam Interferometer with Prism Optics



Schematic diagram of experimental setup at beamline 20XU of SPring-8



Possible Applications

1. **Two-beam Holography,**
2. **Phase Measurement,**
3. **Quantitative Measurement of Spatial Coherence,**
4. **Wavelength Filter, Harmonics Selection .**

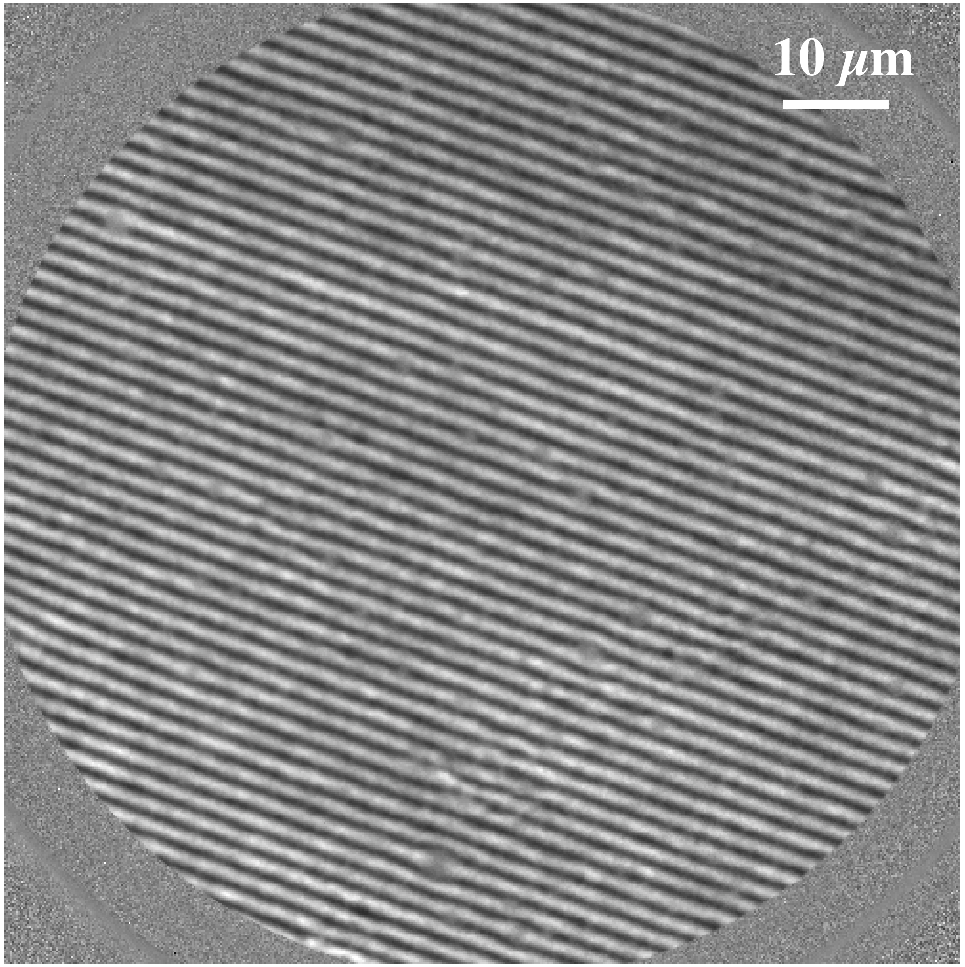
$\Delta\theta = \delta / \tan\theta,$

$\delta = 1.35 \times 10^{-6} \rho \lambda^2.$

$\Delta\theta = 2\ \mu\text{rad},$
for $\rho = 1.2$, $\lambda = 1\text{Å}$, $\theta = 45°$.

$\Delta\theta = 46\ \mu\text{rad},$
for $\theta = 2°$.

Optical system for two-beam interferometer with X-ray prism.



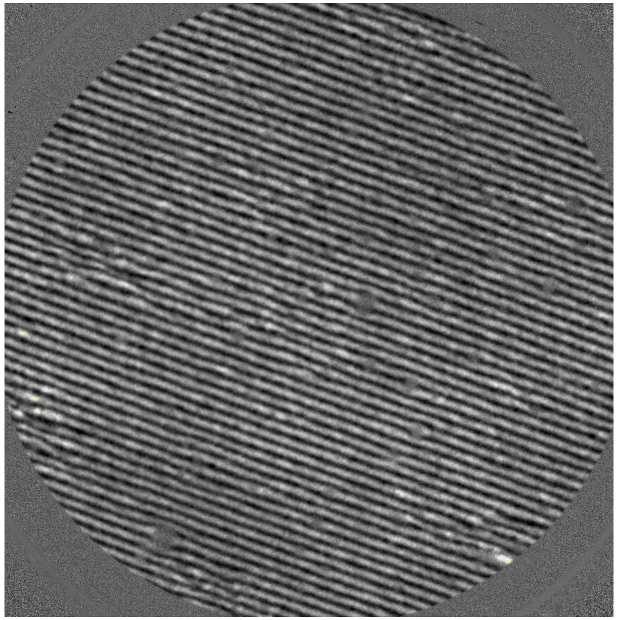
Typical interference fringe patterns measured at an X-ray wavelength of 1 Å.

Beam deflection angle: $\Delta\theta = 44\ \mu$rad.

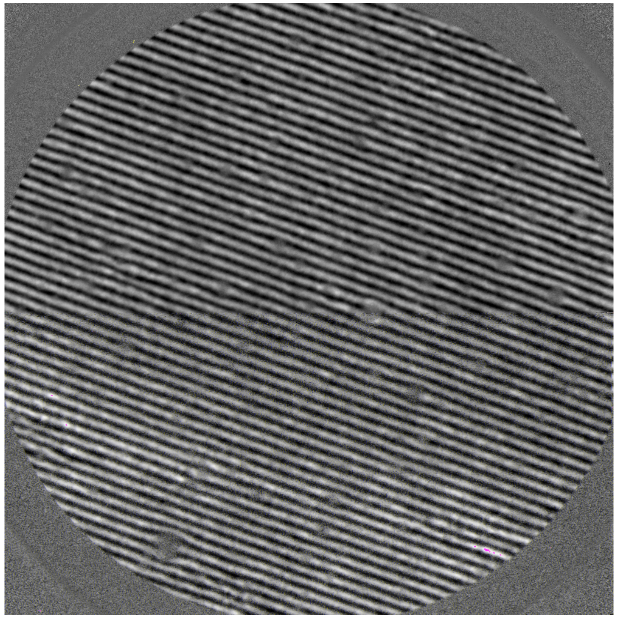
Slit dimensions: 18 μm x 19 μm.

Measured fringe spacing is 2.3 μm.

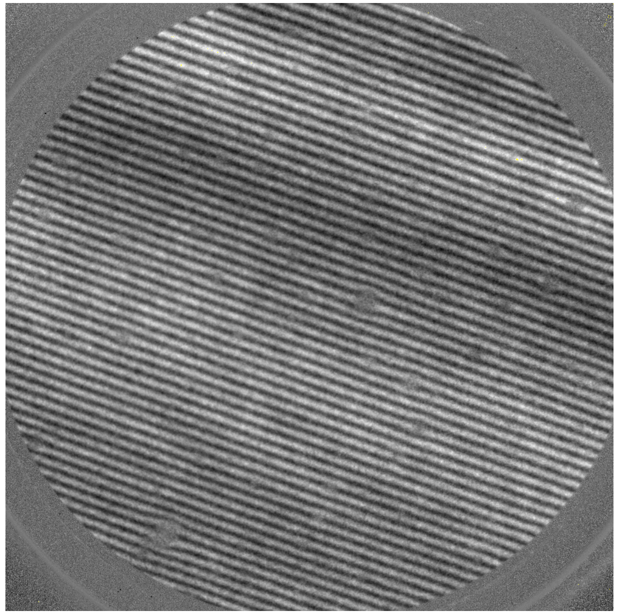
Beam overlap is 300 μm. Exposure time is 60 s.



(a) 1.5 Å and 3 degrees



(b) 1.0 Å and 2 degrees



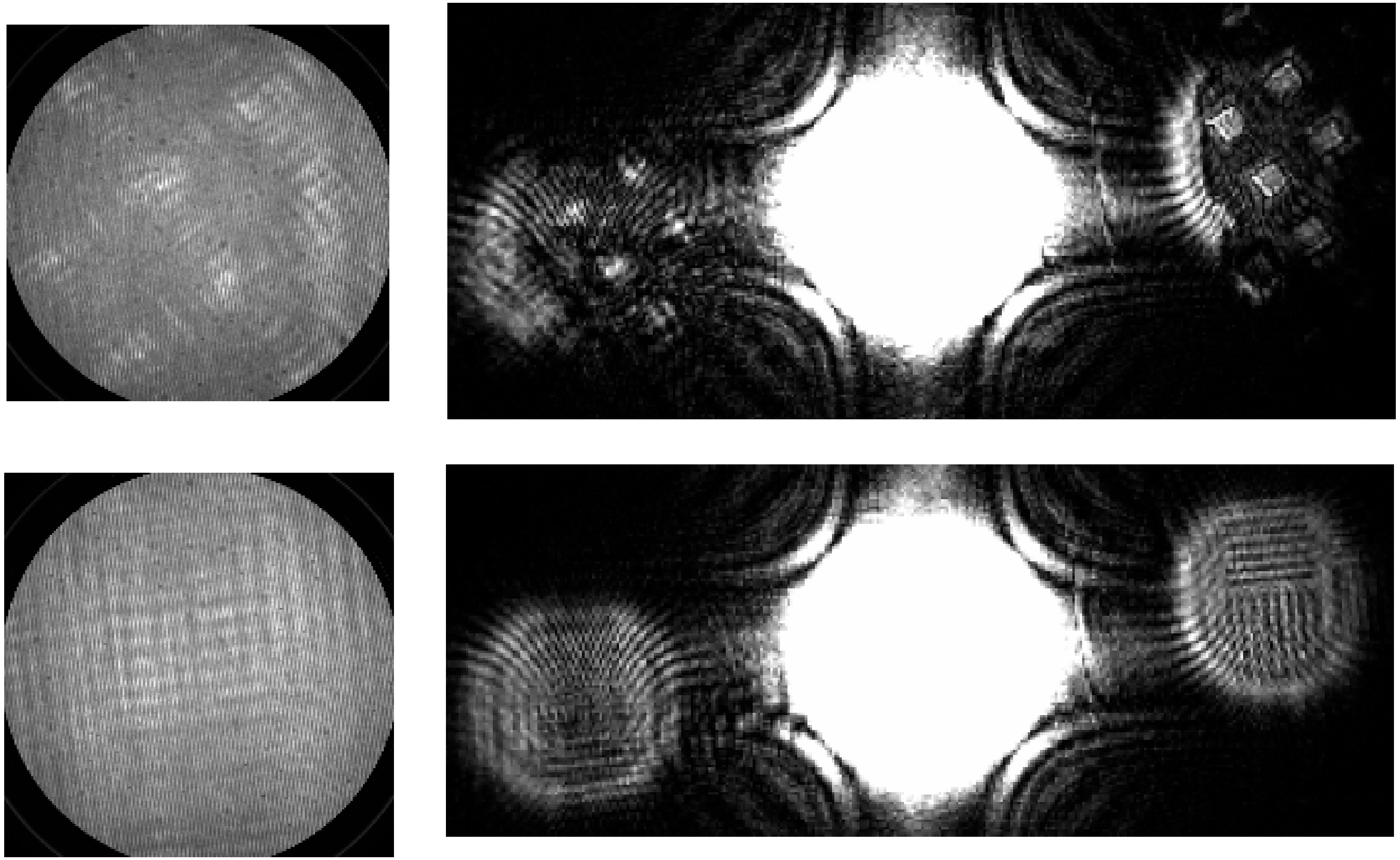
(c) 0.5 Å and 1 degree

Interference Fringes measured with X-ray Zooming Tube.

Prism to detector distance: 6.9 m,
Field of view : 100 μm in diameter,
X-ray wavelength: 0.5, 1.0, and 1.5 Å,
Glancing angle to prism surface: 1.0, 2.0, and 3.0 degrees.

Cu grid mesh
64 μm pitch

Test Patterns
5 μm L/S

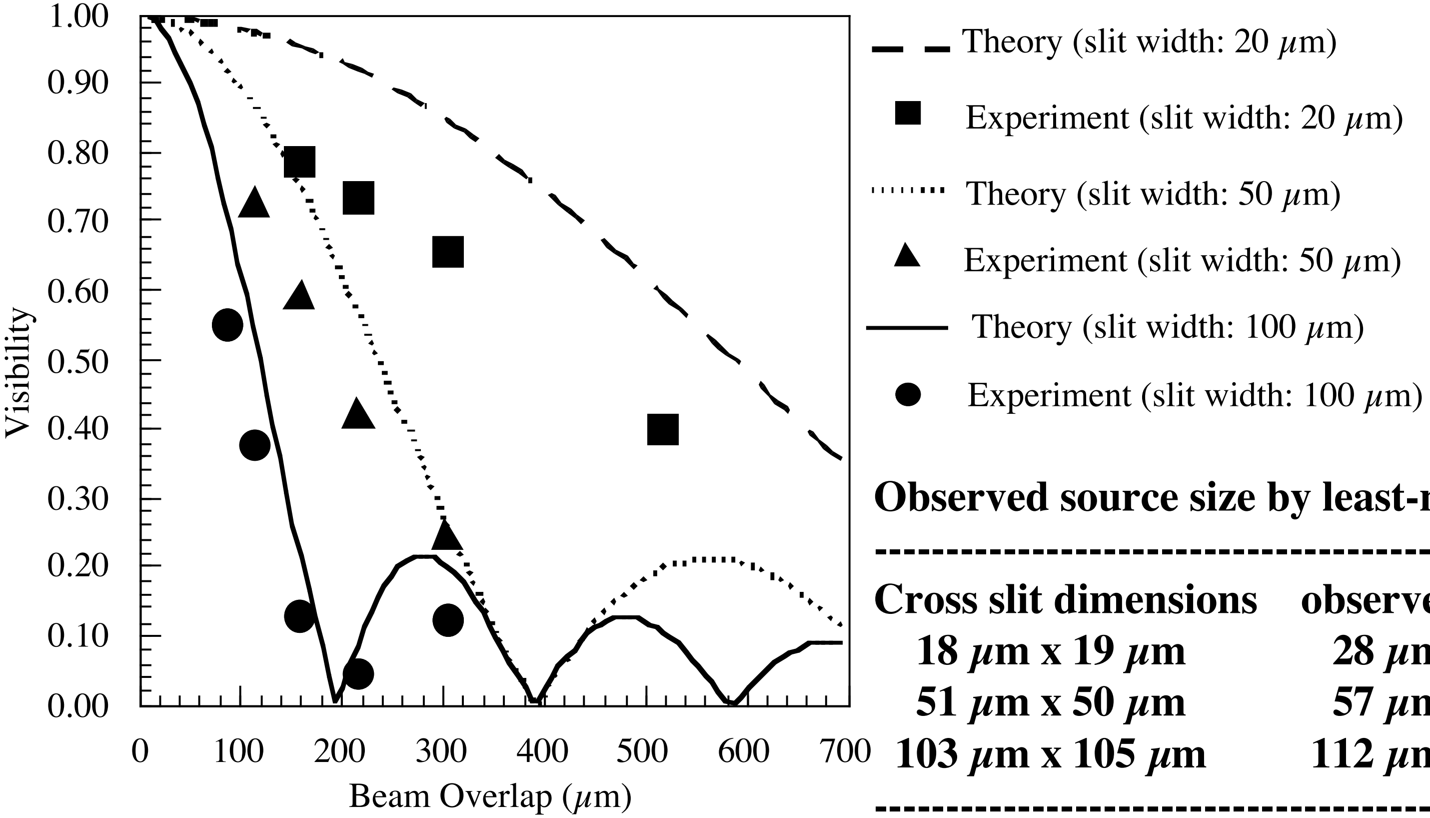


Hologram

Reconstructed Image

Leith-Upatnieks Type Two-beam Holography

λ = 1.0 Å, Sample to image detector: 6.7 m



Observed source size by least-mean-square curve fit

Cross slit dimensions	observed source dimension
18 μm x 19 μm	28 μm
51 μm x 50 μm	57 μm
103 μm x 105 μm	112 μm

Visibility of interference fringes.

Solid squares, circles and triangles represent experimental data,
Solid line, dotted line and dashed line are respective theoretical curves
for source size of 100 μm, 50 μm, and 20 μm.

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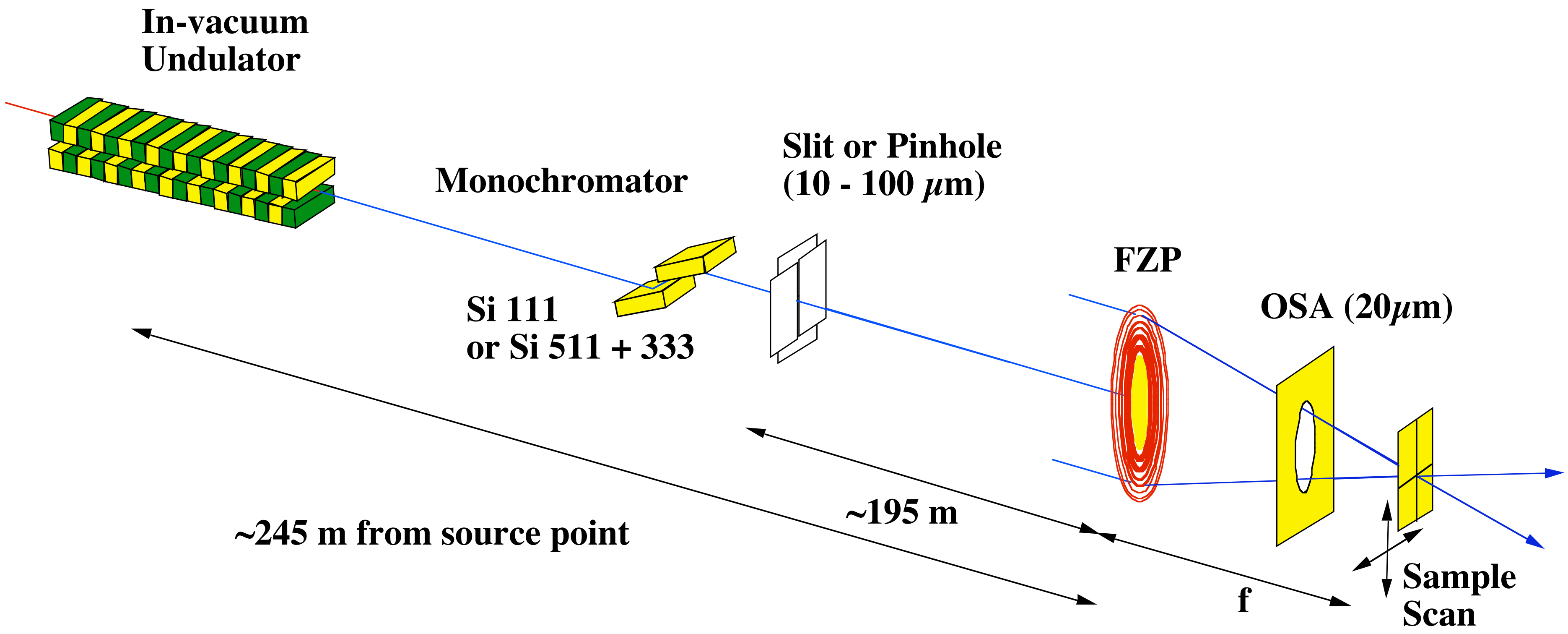
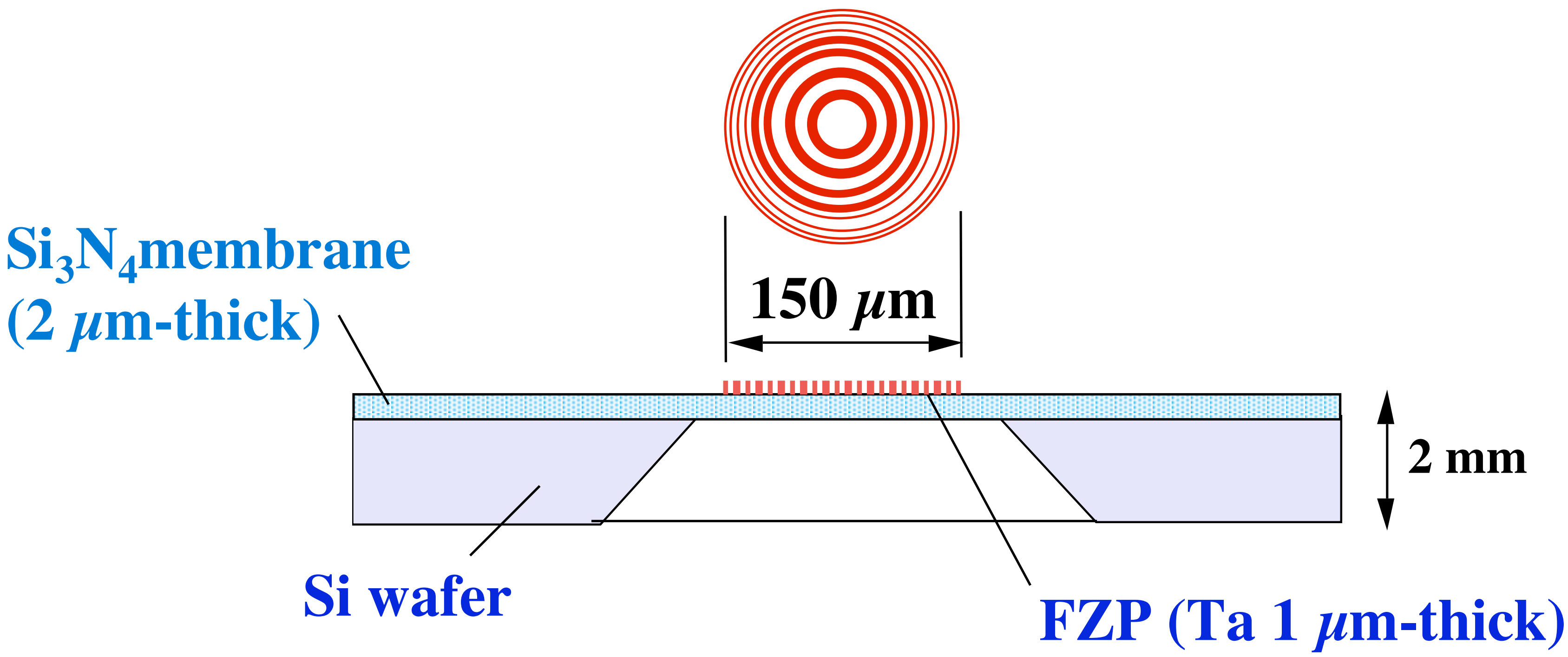


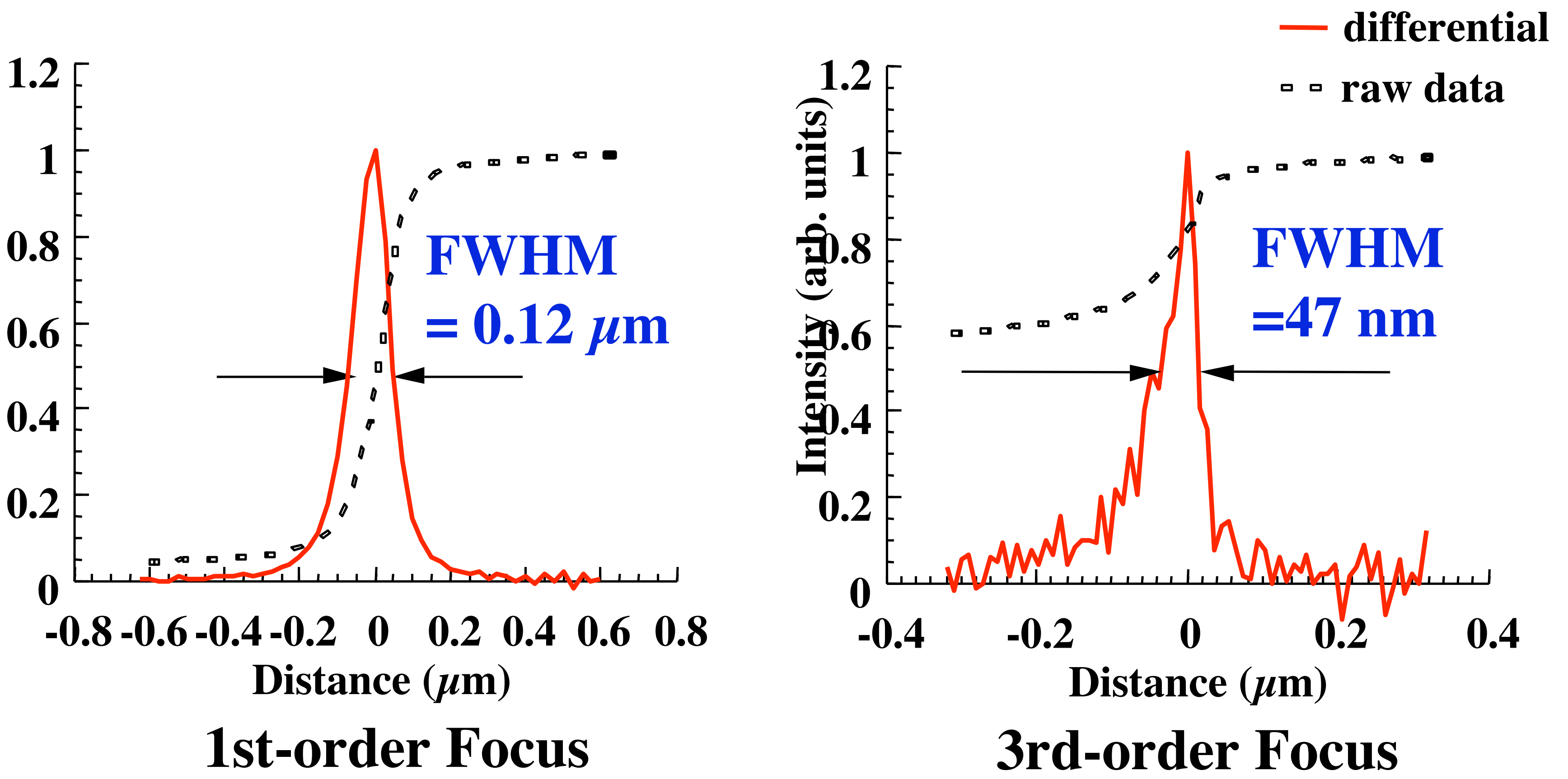
Experimental Setup of X-ray Microbeam/Scanning Microscopy at BL20XU

Specification of Fresnel zone plate

Diameter: 150 μm, Designed focal Length: 100 mm at 8 keV,
Outermost zone width (d_N): 0.1 μm.
Diffraction limit ($=1.22d_N$): 0.12 μm, numerical aperture: 7.5×10^{-4} at 8 keV,
Zone material: Ta, 1 μm-thick,
Supporting membrane: Si_3N_4 , or SiC, 2 μm-thick.
Fabrication method: electron-beam lithography technique at NTT-AT

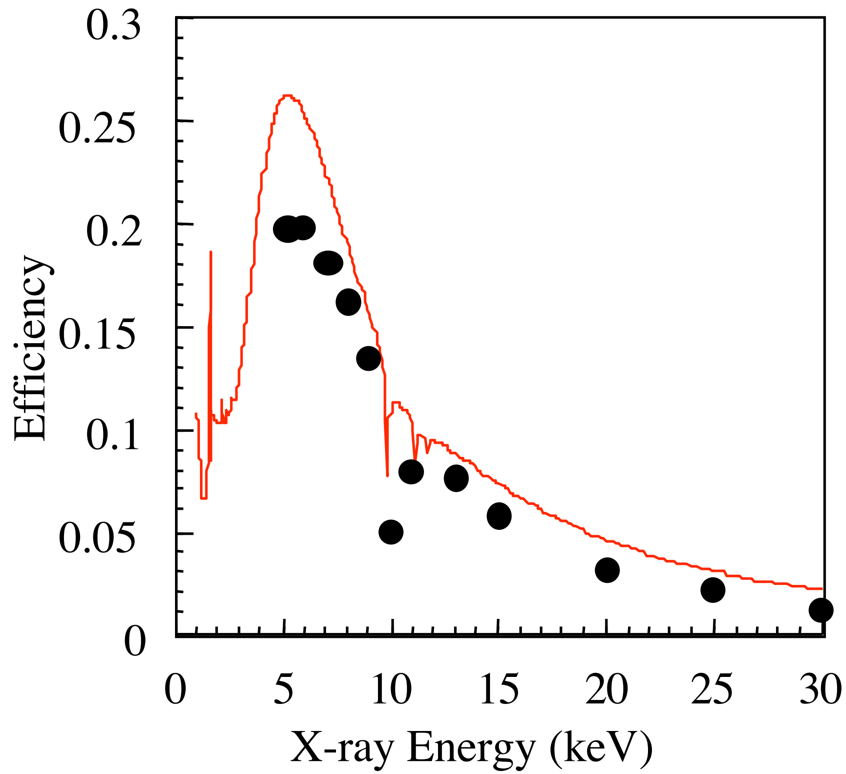


Schematic Drawing of Zone Plate Structure



Focused Beam Profile Measured by Knife-edge Scan

FZP: Ta 1 μm-thick,
Outermost Zone Width: 0.1 μm,
EB-lithography at NTT-AT,
Focal Length: 100 mm @8 keV.



Diffraction Efficiency of Ta-FZP

Closed Circle: Experimental Results,

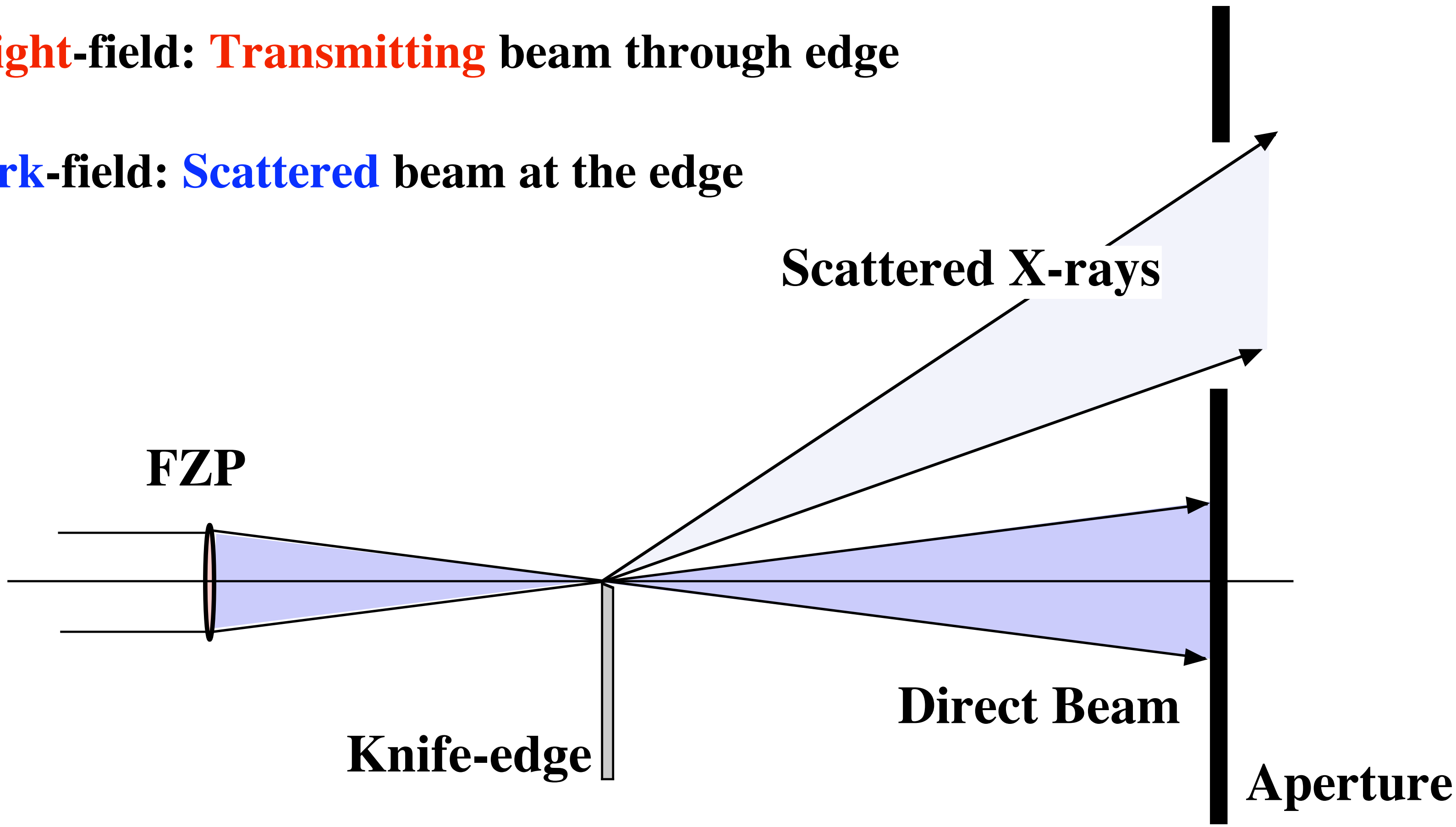
Solid Line: Calculated Efficiency assuming the Thickness of 1μm.

Total flux of microbeam: 10^9 photons/s,

Focused beam size: 0.12 μm.

Bright-field: Transmitting beam through edge

Dark-field: Scattered beam at the edge

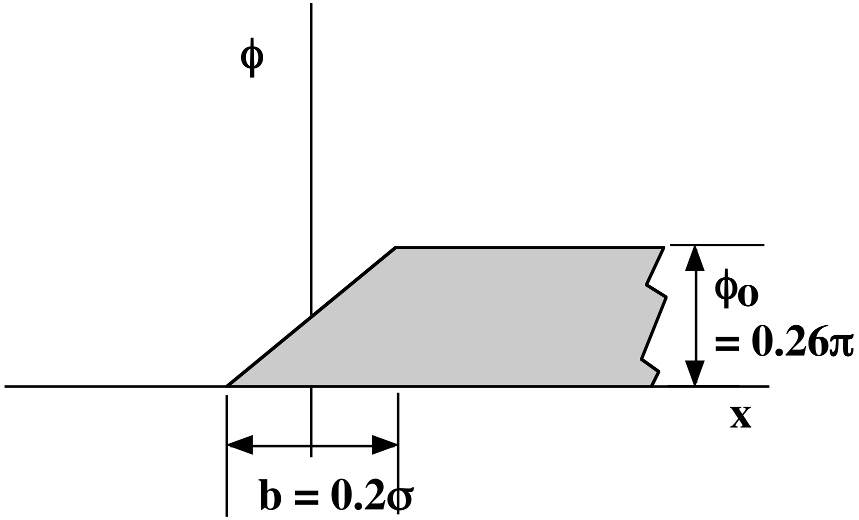


Dark-field Knife-edge Scan Method for Focus Test

Primary beam (direct beam) is cut off with an aperture in front of X-ray detector.
The scattered X-rays are selected by the aperture.

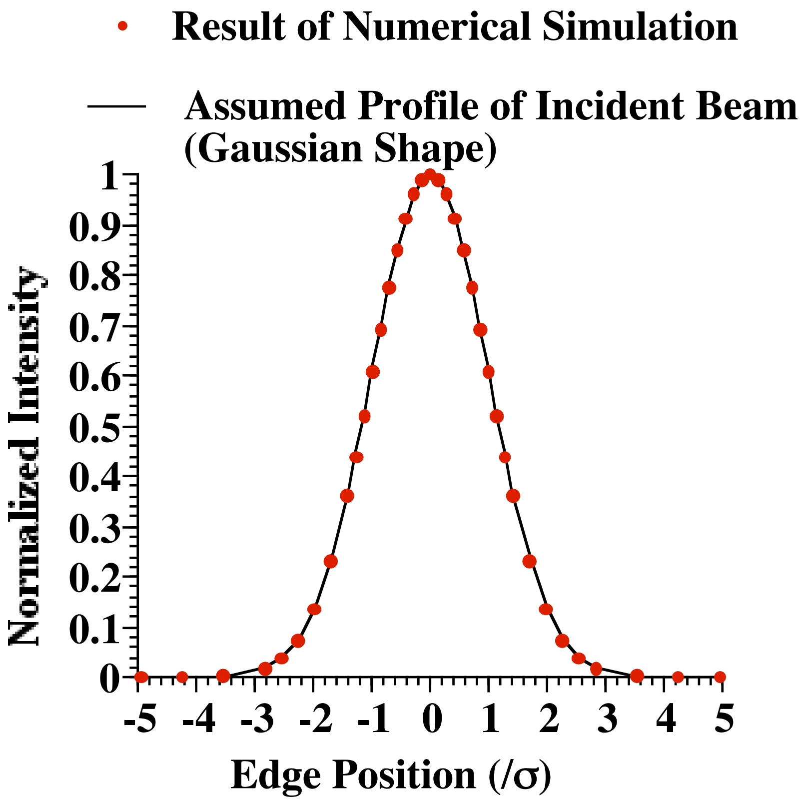
Advantages of dark-field method:

1. **Thin and phase** object as knife-edge,
2. **No differential** processing,
3. Precise measurement.



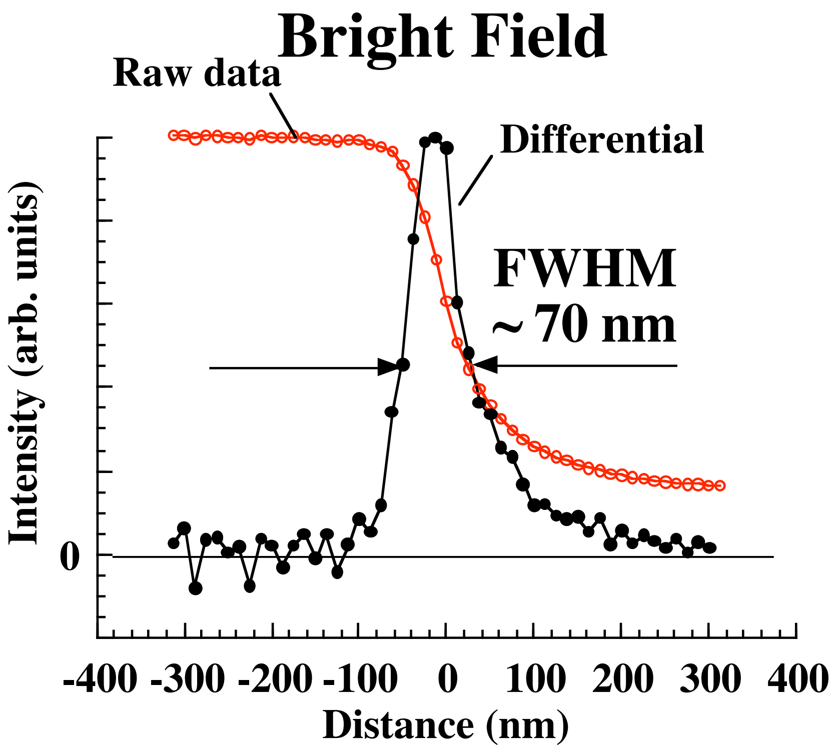
Assumed Cross-section of Knife-edge.

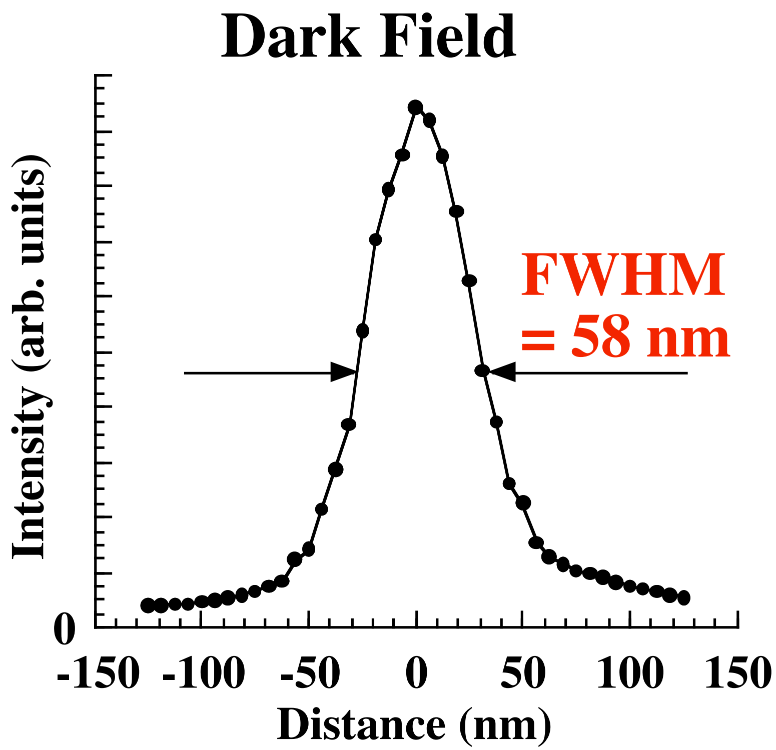
Tapered edge with width of *b*,
and phase shift of ϕo.



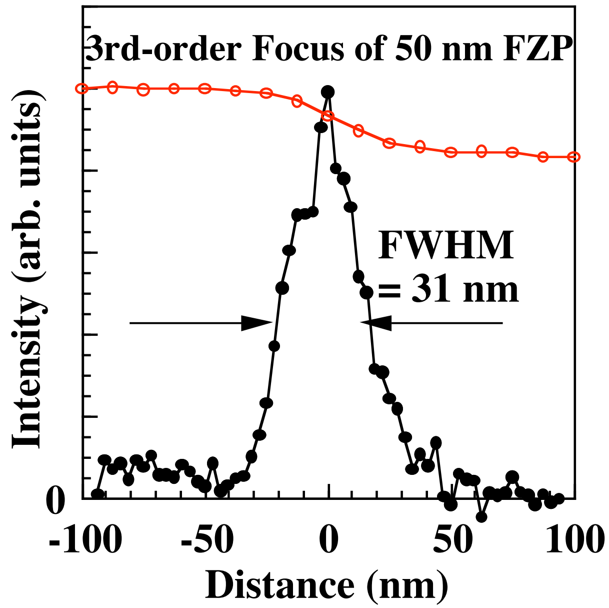
Result of Numerical Simulation

Simulation of Dark-field Knife-edge Test

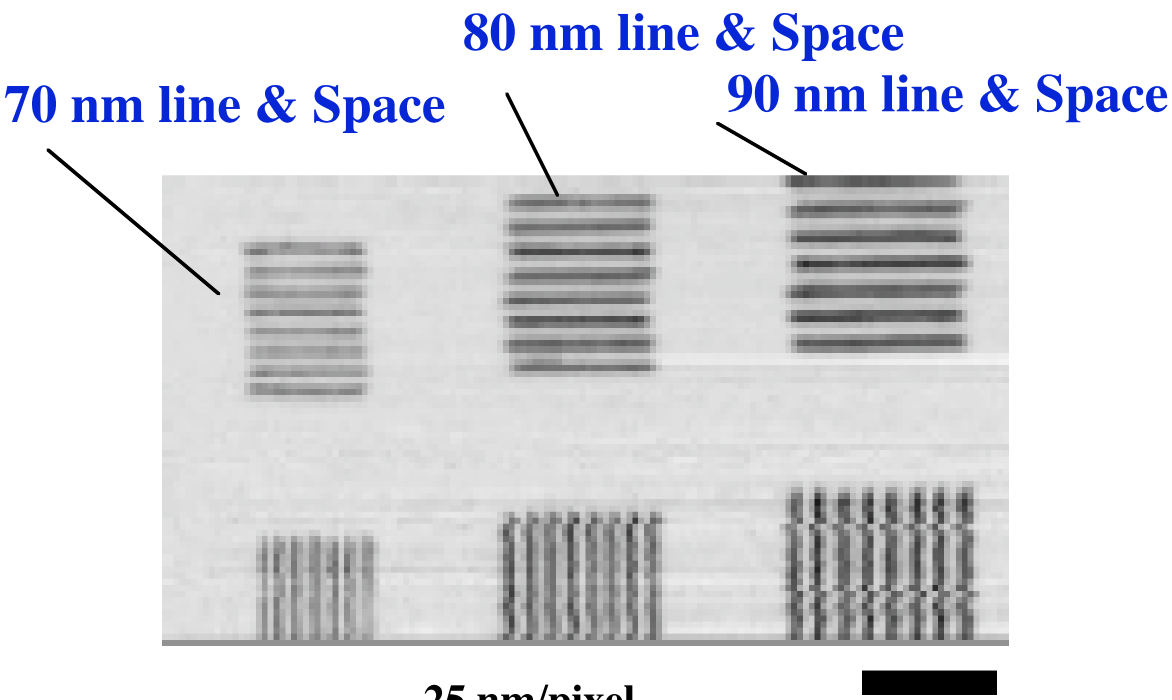




Result of Knife-edge Test
Bright-field and Dark-field method

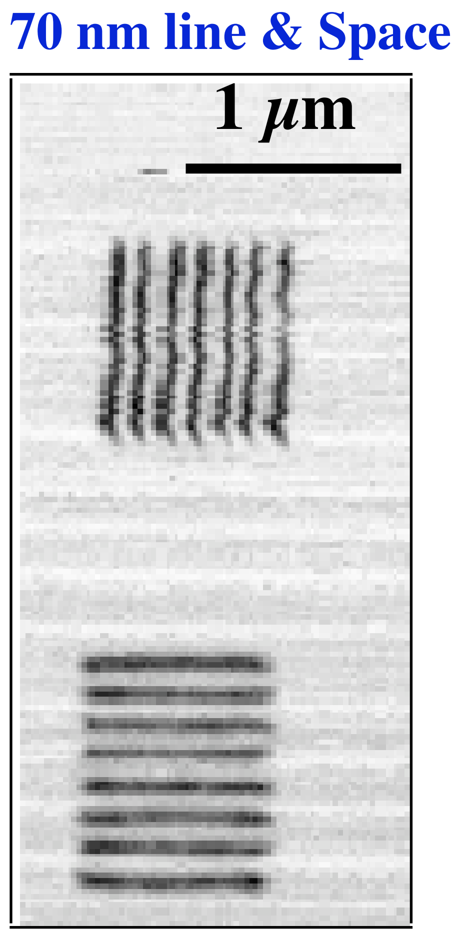


Outermost Zone Width: 50 nm, 1st-order Focus,
f **= 80 mm @8 keV,**
Knife-edge: gold wire of 50 μm diameter.

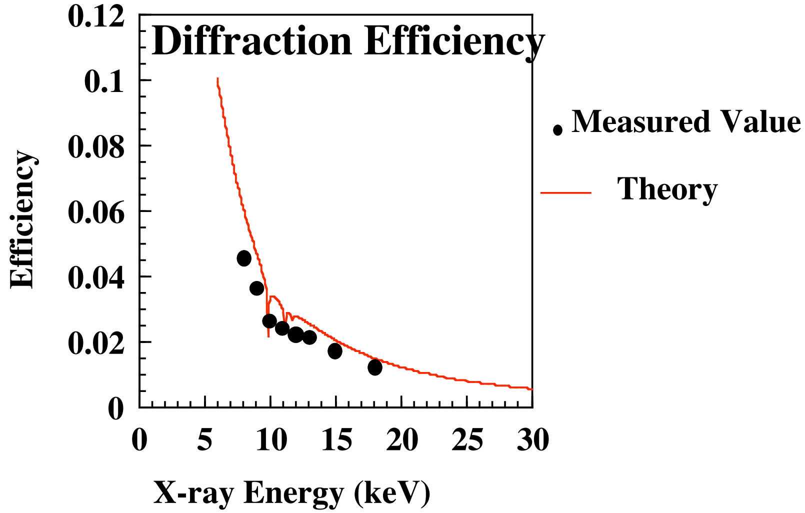


1 μm

25 nm/pixel,
Dwell Time: 0.5 s.

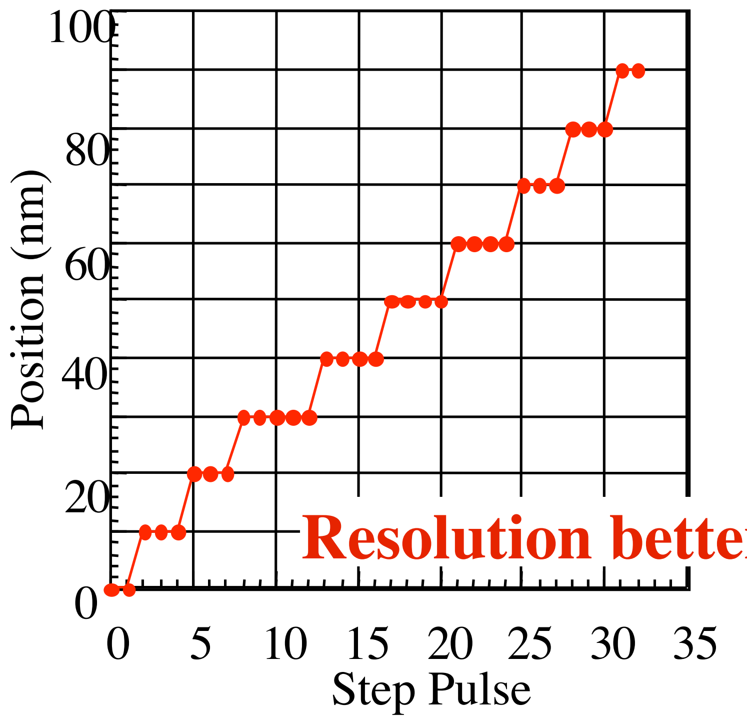


76 x 163 pixels,
12.5 nm/pixel,
Dwell Time: 0.5 s.

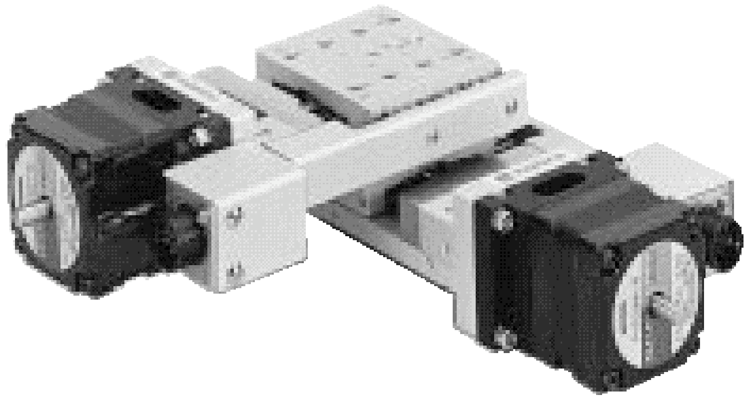


Scanning Microscopy

Test Patterns (Ta 500 nm-thick),
X-ray Energy: 8 keV.



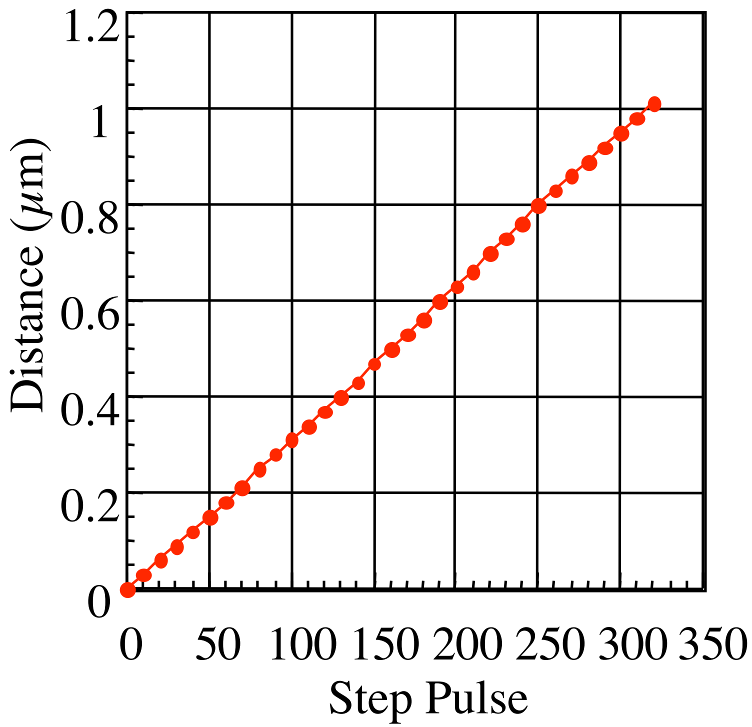
Resolution better than 10 nm

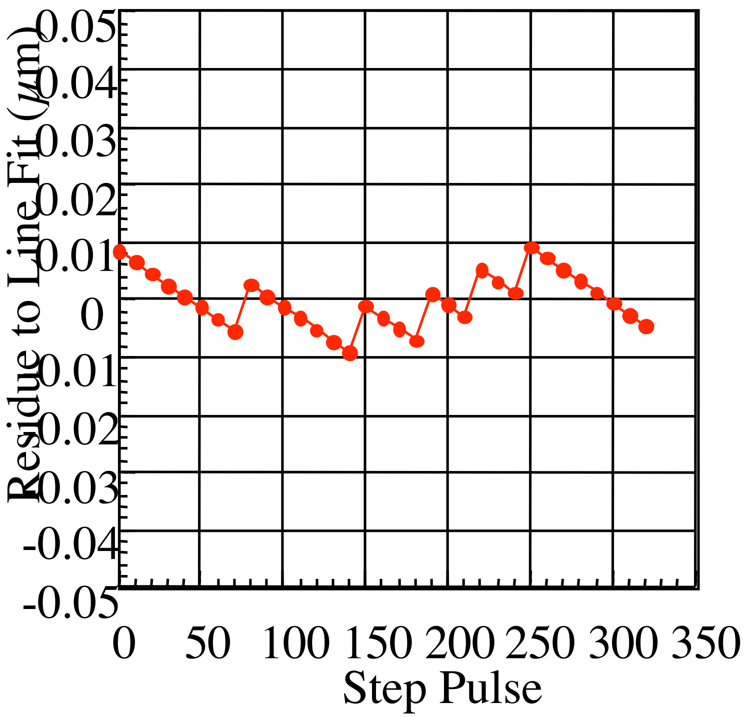


1/320 μm/step = 3.125 nm/step

Resolution Test of Linear Translation Stage

Kohzu Seiki, type YA-05-14.
Stepping Motor, Oriental Motor PX535MH-B.
Motor Driver, Melec, micro-step drive H-583.
Position Monitor, KEYENCE LC2420.

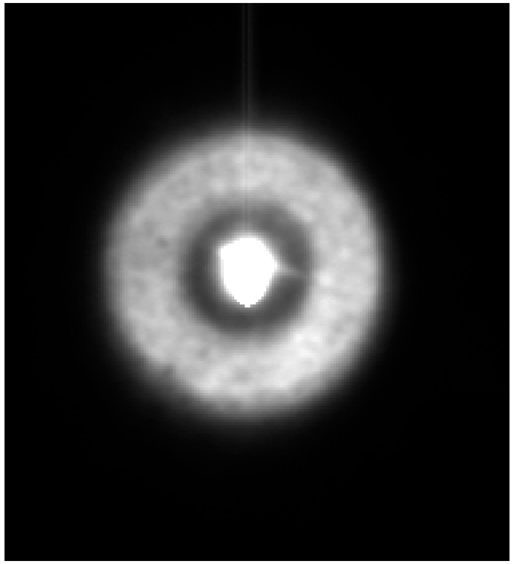




Test Result of Linear Translation Stage

Kohzu Seiki, type YA-05-14.
Stepping Motor, Oriental Motor PX535MH-B.
Motor Driver, Melec, micro-step drive H-583.
Position Monitor, KEYENCE LC2420.

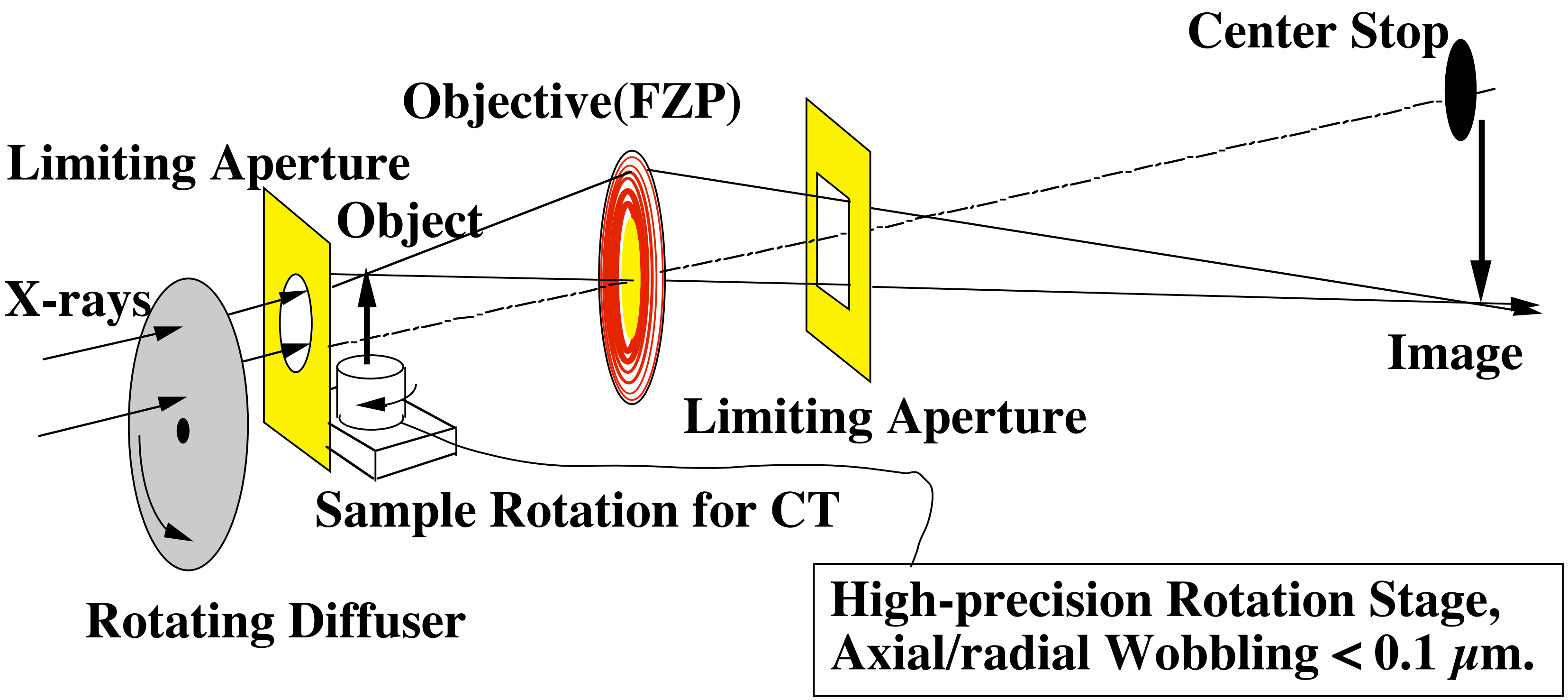
Lifetime of Ta-FZP for radiation damage
~ 3 days in air,
>> 1 month in vacuum or in He.



Irradiation in air

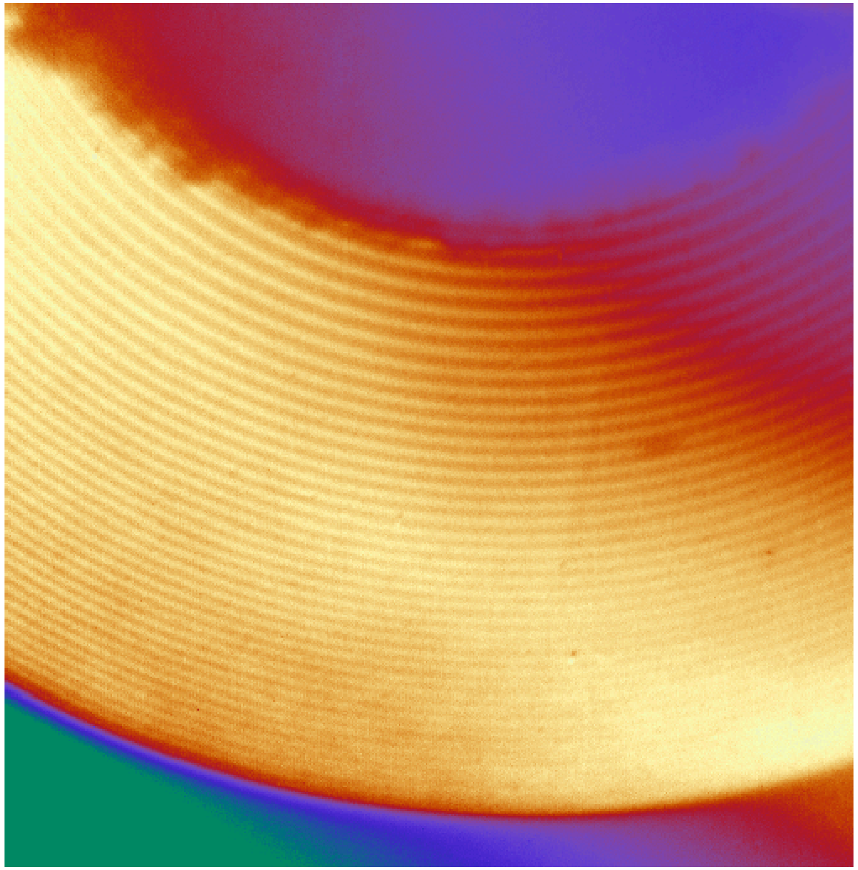
FZP: 100 μm diameter,
0.25 μm outermost zone width,
X-ray energy: 10 keV @BL47XU,
Flux density: ~ 5 x 10^{13} photons/s/mm,
Total flux: ~ 10^{17} photons

Damage problem is solved at present!



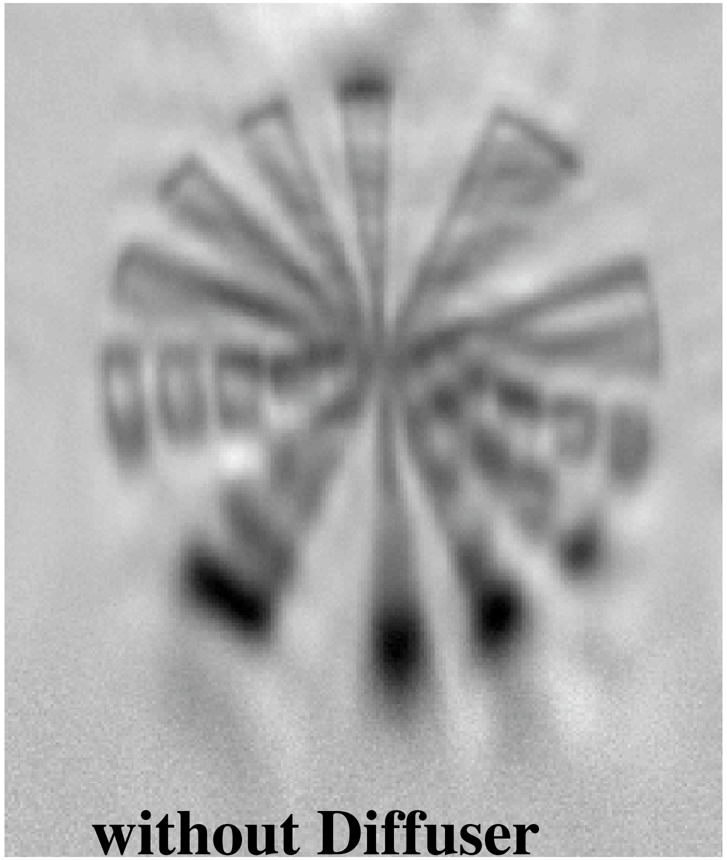
Schematic Diagram Experimental Setup
for Imaging Microscopy and Micro-tomography

@ BL47XU

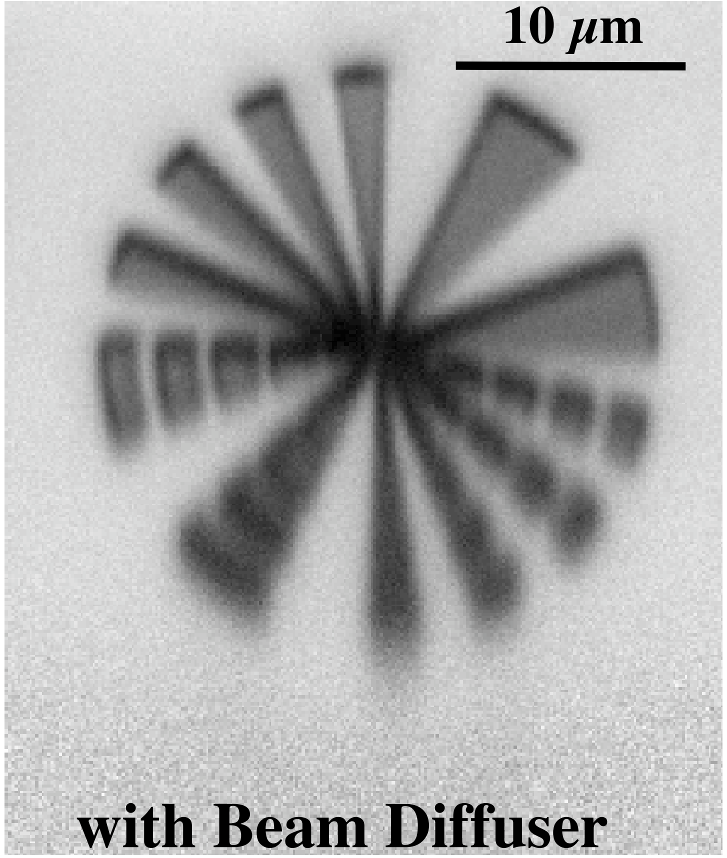


Imaging Microscopy

Objective &Sample:FZP with 0.25 μm outermost zone width,
X-ray Energy: 8 keV.



- Coherent Illumination -

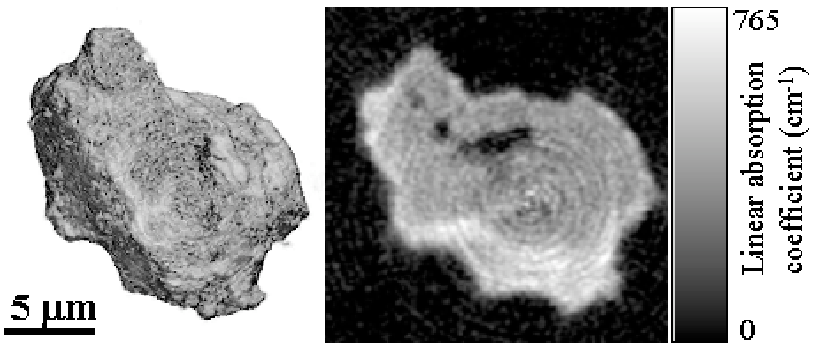


- Incoherent Illumination-

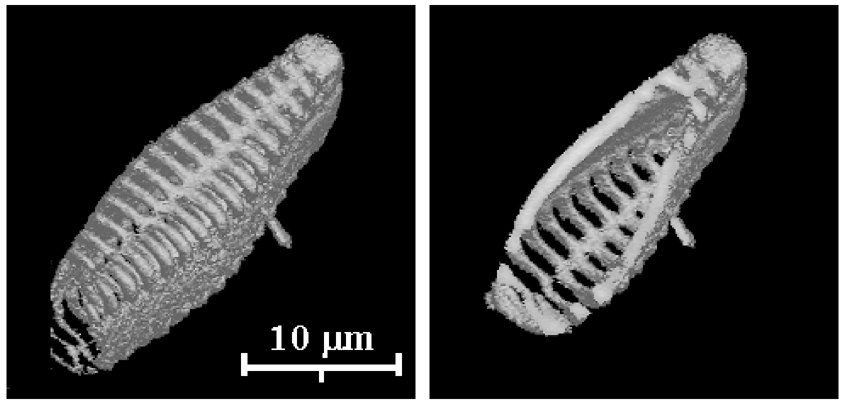
Imaging Microscopy with FZP Objective
- Effect of Beam Diffuser -

X-ray energy: 8 keV

Control of coherence is important!



Stony Meteorite Allende
8 keV, x7.61, BM3(x10), voxel size 0.13 μm.
100 projection, exposure time:15 s/projection.



Diatom "Achnanthidium lanceolata”
8 keV, x10, BM3(x10), voxel size 0.1 μm.
360 projection, exposure time: 60 s/projection.

X-ray Micro-tomography using Imaging Optics with Fresnel Zone Plate Objective

Low-emittance SR source is not suitable for imaging microscopy, because of

high spatial coherence:

Small source size (~ 10 μm vertical x 100 μm horizontal),
Small divergent angle (~ 10 μrad).

Critical illumination with simple condenser lens:

F-number matching --> small field of view (< a few μm),
Coherent illumination --> Speckle noise.

Critical illumination and Köhler illumination

(best optics for imaging microscopy)

Critical illumination:

Demagnified image of source at the object plane.,
Each point of source corresponds to each point of field of view,
Not suitable for 3rd generation SR source.

Köhler's illumination:

Infinite focus,
Each points at source to each angle of illuminating beam.

Illumination Optics for Imaging Microscopy

First experiment on imaging microscopy at SPring-8:

Parallel beam illumination

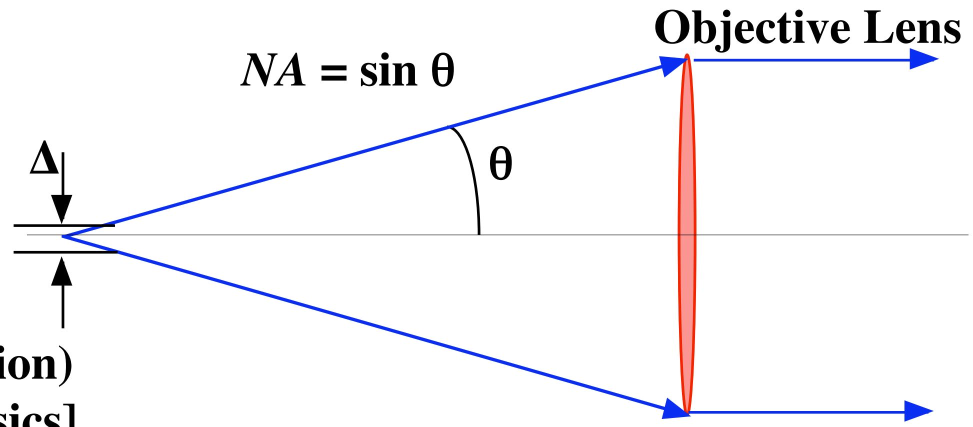
--> Edge-enhancement artifact,
Strong speckle noise.

2nd Step:

Partial Coherent illumination by diffuser

--> Less artifacts, and no speckles,
Weak edge-enhancement,
Nonuniform imaging properties in the field of view,
Asymmetric feature of imaging properties.
(off-axial illumination)

Need of Condenser Optics for Imaging microscopy.



Rayleigh's criterion (incoherent condition)
[uncertainty principle in quantum physics]

$$\Delta = 0.61\ \lambda/NA,$$

Parallel beam illumination:

$$\Delta = 0.82\ \lambda/NA,$$

With condenser optics of 1.5NA:

$$\Delta = 0.57\ \lambda/NA,$$

NA: **numerical aperture of objective lens.**

$$NA = \sin\theta.$$

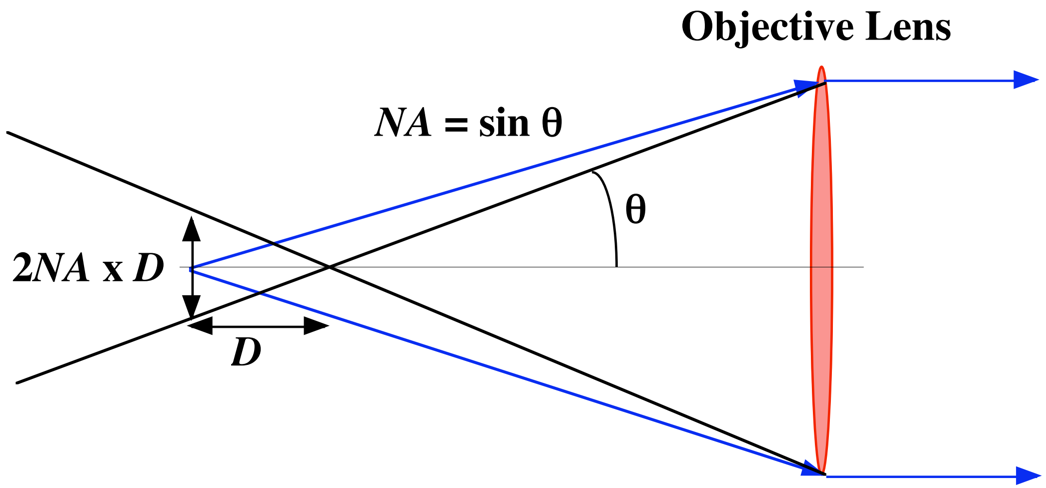
Useful formula:

$$\Delta = 1.22\ dr_N$$

dr_N: **Outermost zone width**

Example:
$dr_N = 100$ nm,
$\Delta = 122$ nm.
$NA = 7.75 \times 10^{-4}$ at 8 keV.
(8 keV = 1.55 Å)

Spatial Resolution of FZP Microscope



Geometrical defocusing:

$$2D \times NA$$

Limit of defocusing:
Diffraction-limited resolution = Geometrical defocusing

$$2D \times NA = 0.61\ \lambda/NA,$$

Depth of focus (tolerance of sample thickness):

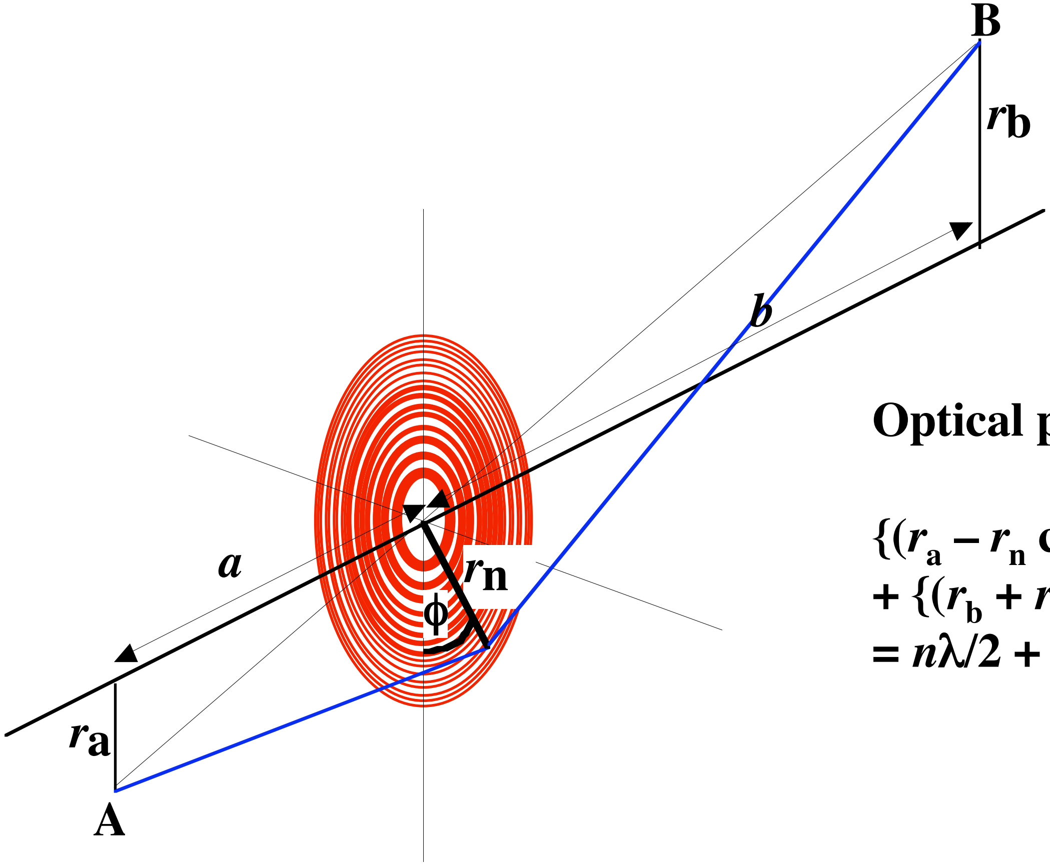
$$2D = 0.61\ \lambda/NA2.$$

In tomography measurement,
Depth of focus > Sample diameter.

Example:
Δ = 122 nm (dr_N = 100 nm),
$NA = 7.75 \times 10^{-4}$ at 8 keV.

$2D = 157\ \mu m$.

Depth of Focus



Optical path description:

$$\{(r_a - r_n \cos\phi)^2 + (r_n \sin\phi)^2 + a^2\}^{1/2} + \{(r_b + r_n \cos\phi)2 + (r_n \sin\phi)^2 + b^2\}^{1/2} = n\lambda/2 + (a^2 + r_a^2)^{1/2} + (b^2 + r_b^2)^{1/2},$$

Second order approximation and the Rayleigh's quarter wavelength rule:

$$|\{- r_a r_n \cos\phi/a + r_b r_n \cos\phi/b\} + 1/2\{ r_n^2/a + r_n^2/b - n\lambda\} - 1/8\{(r_a^2 - 2r_a r_n \cos\phi + r_n^2)^2/a^3 - r_a^4/a^3 + (r_b^2 + 2r_b r_n \cos\phi + r_n^2)^2/b^3 - r_b^4/b^3\}| < \lambda/4.$$

Aberration Theory of FZP Microscope by Wave Optics

Depth of focus (tolerance of sample thickness for tomography):

$$2D = 0.61\,\lambda/NA^2,$$

$NA \sim r_N/f$, $D \sim r_a$ ----> $r_a r_N^2/f^2 < 0.3\lambda$

$r_N/f \ll 1$, $r_a/f \ll 1$ **for hard -X-ray FZP.**

Other wavefront aberrations:

$3r_a^2 r_N^2/f^3 < \lambda,$

$2r_a r_N^3/f^3 < \lambda,$

$1/2 r_N^4/f^3 < \lambda$

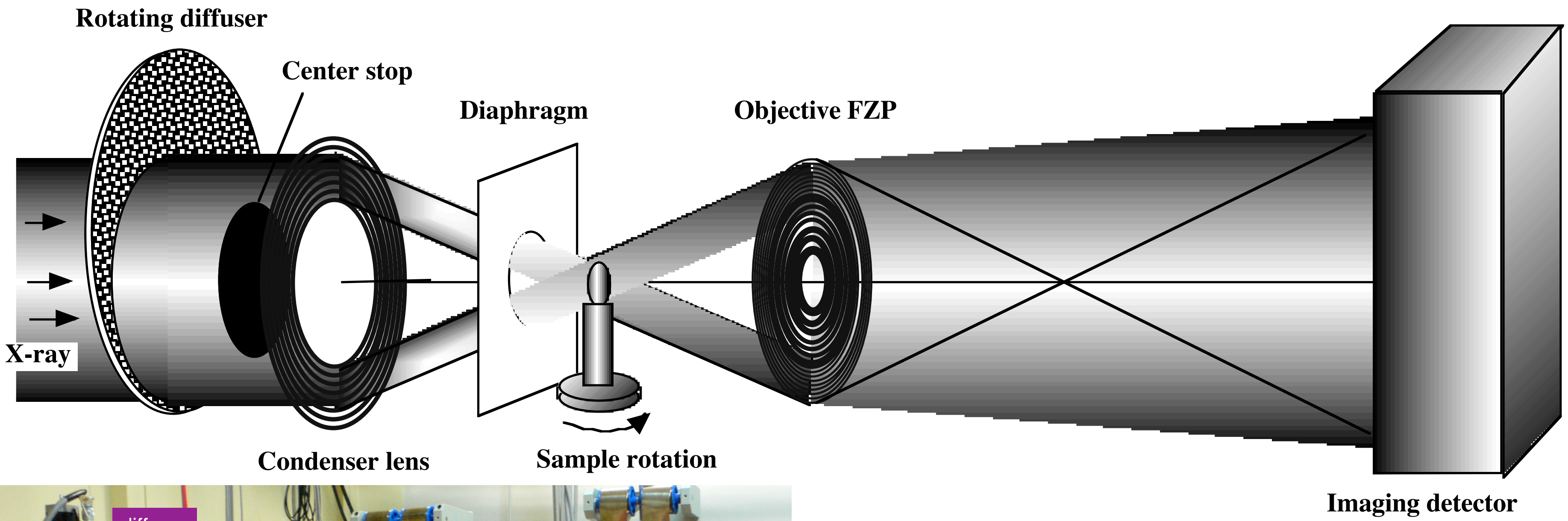
Chromatic aberration:

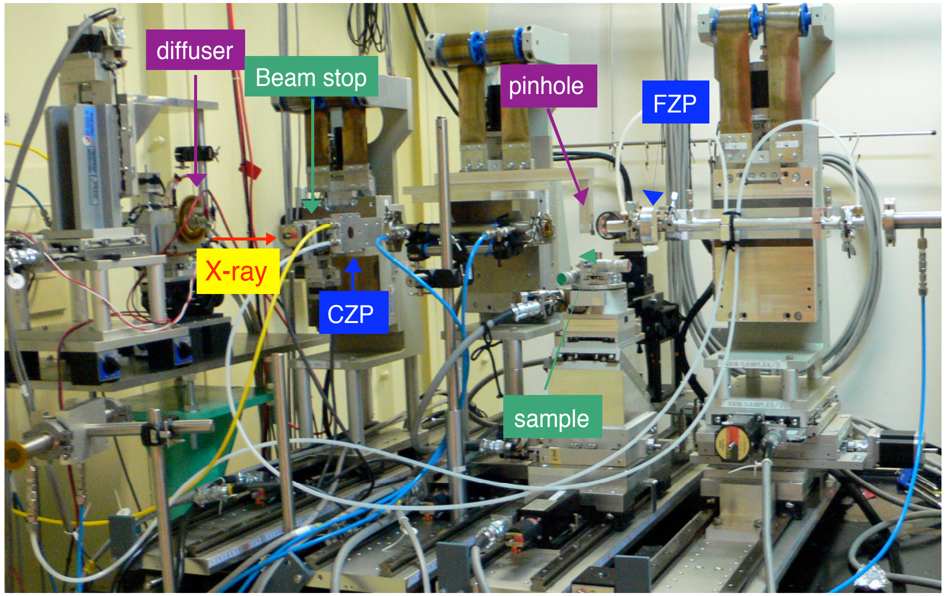
$\Delta\lambda/\lambda < 0.61/N$, (*N*: total zone number).

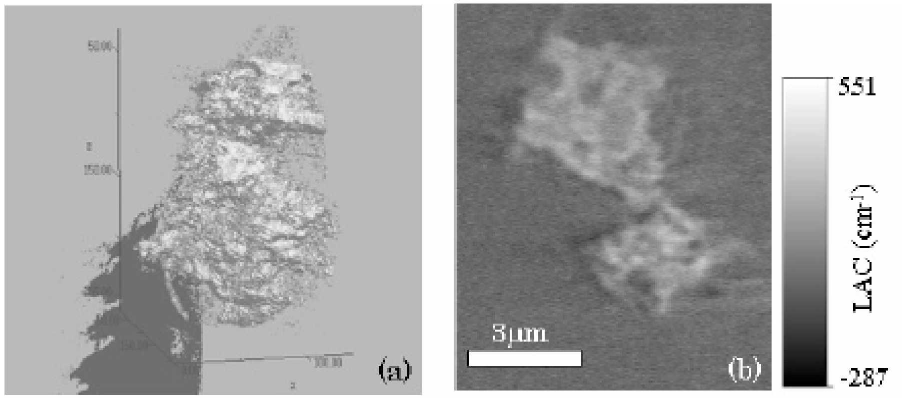
$\Delta\lambda/\lambda \sim 10^{-4}$ (crystal monochromator, Si 111)

Example:
$f = 100$ mm at 8 keV,
$r_N = 77.5\ \mu$m,
$N = 388$,
$NA = 7.75 \times 10^{-4}$ at 8 keV.
($\lambda = 1.55$ Å)
M ~ 70

Most serious aberration is Depth of Focus!







CT image of IDP (L2008D3 #17). (a) three dimensional reconstruction from CT images. (b) sagital slice derived from three dimensional reconstructed image.

Hard X-ray Imaging Micro-tomography

Fresnel zone plate = Chromatic aberration,

Requirement on monochromaticity for Fresnel zone plate
~ Number of Fresnel zone.

---> $\Delta\lambda/\lambda < 1/N$ (number of Fresnel zone)

$N \sim$ or > 100, (requirement for natural lens approximation).

↓

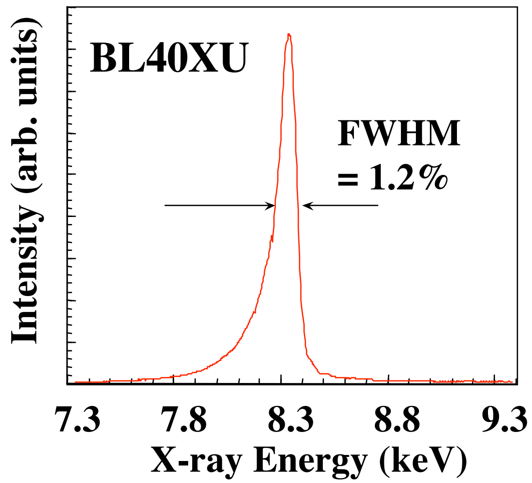
$\Delta\lambda/\lambda \sim 10^{-4}$ with crystal monochromator,
too narrow! ---> loss of photon flux.

↓

Use of direct undulator radiation,
$\Delta\lambda/\lambda \sim 100$.
----> High flux microbeam,
Short Exposure Time.

BL40XU of SPring-8 (High Flux Beamline)

1. Undulator radiation without monochromator,
   $\Delta\lambda/\lambda \sim 1.2\%$ @ ε = 3 nm rad
2. Helical Undulator --> Suppression of higher order,
3. Condenser Optics: K-B mirror

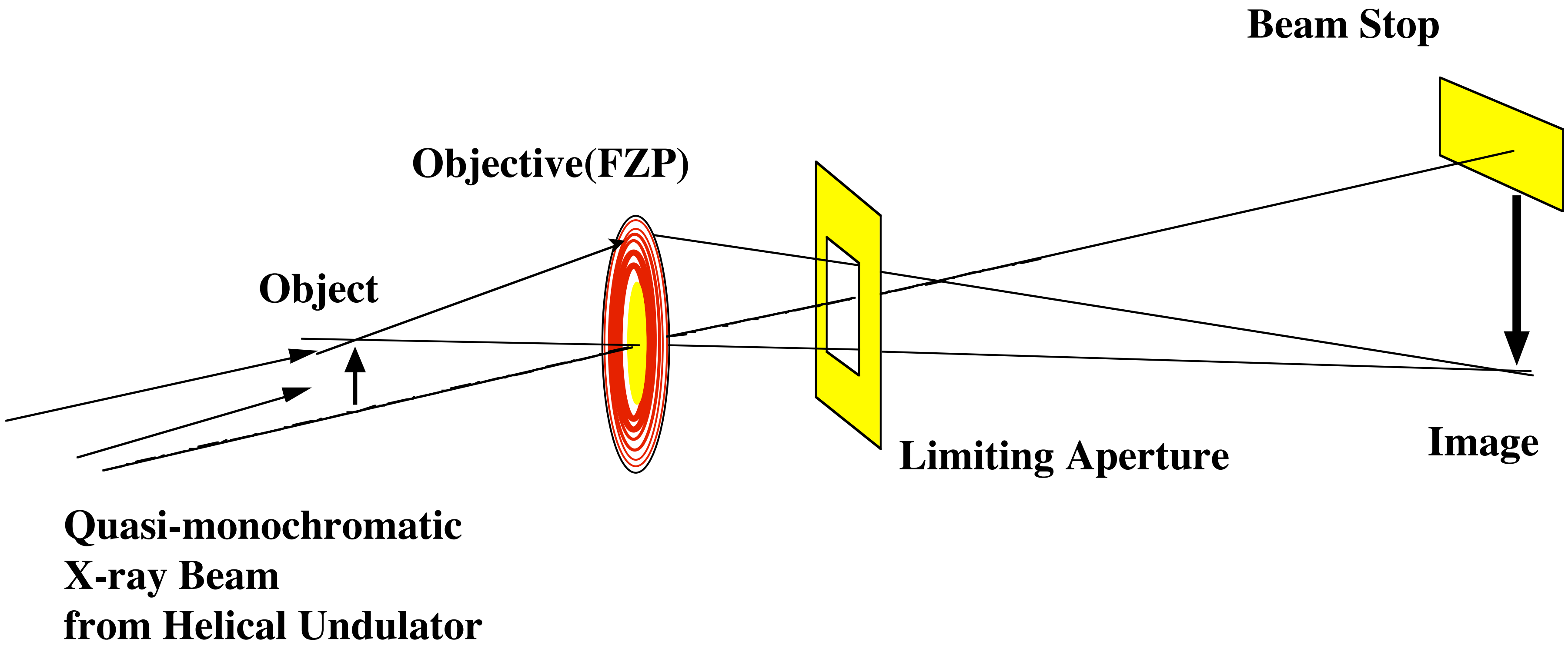


Measured Spectra of Undulator Radiation
Front-end Slit Aperture:
15 μrad (horizontally) x 5 μrad (vertically)

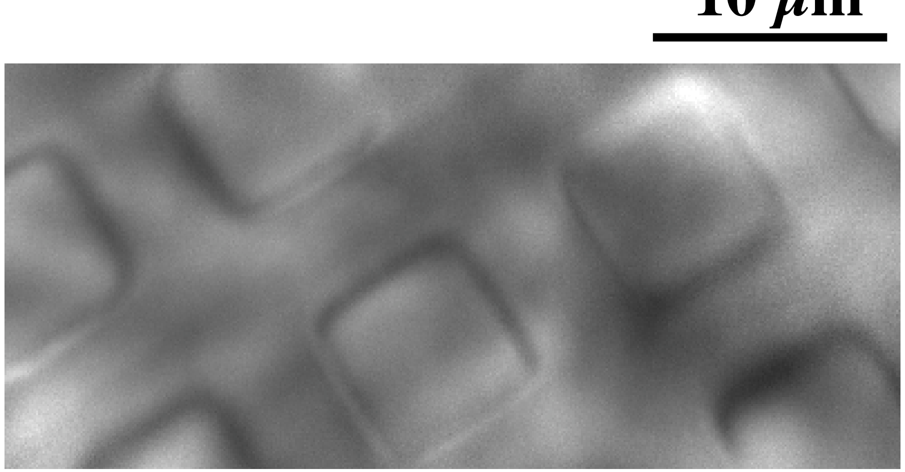
↓

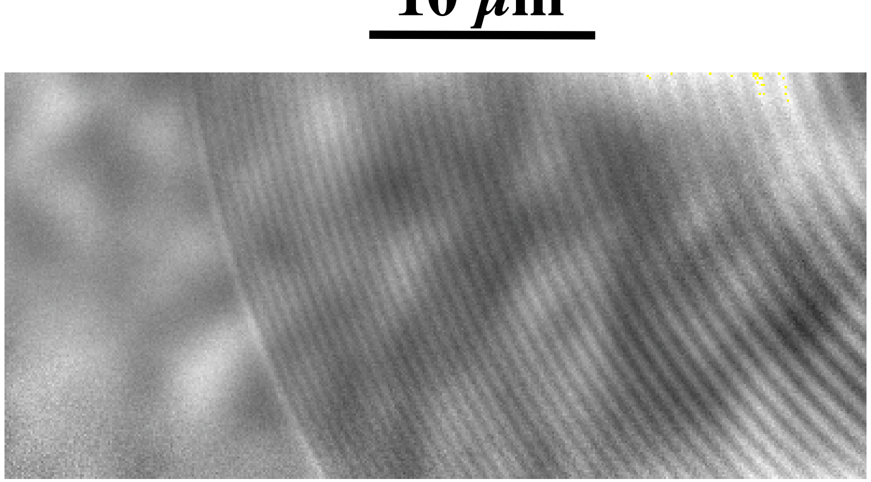
Available flux ~ 100 times that at conventional beamlines
(undulator beamlines with crystal monochromator.

X-ray Microbeam & Imaging Microscopy with
Sub-micron resolution
and high flux!
(~ 100 times, compared with conventional beamlines)



Experimental Setup for Imaging Microscopy
at BL40XU SPring-8





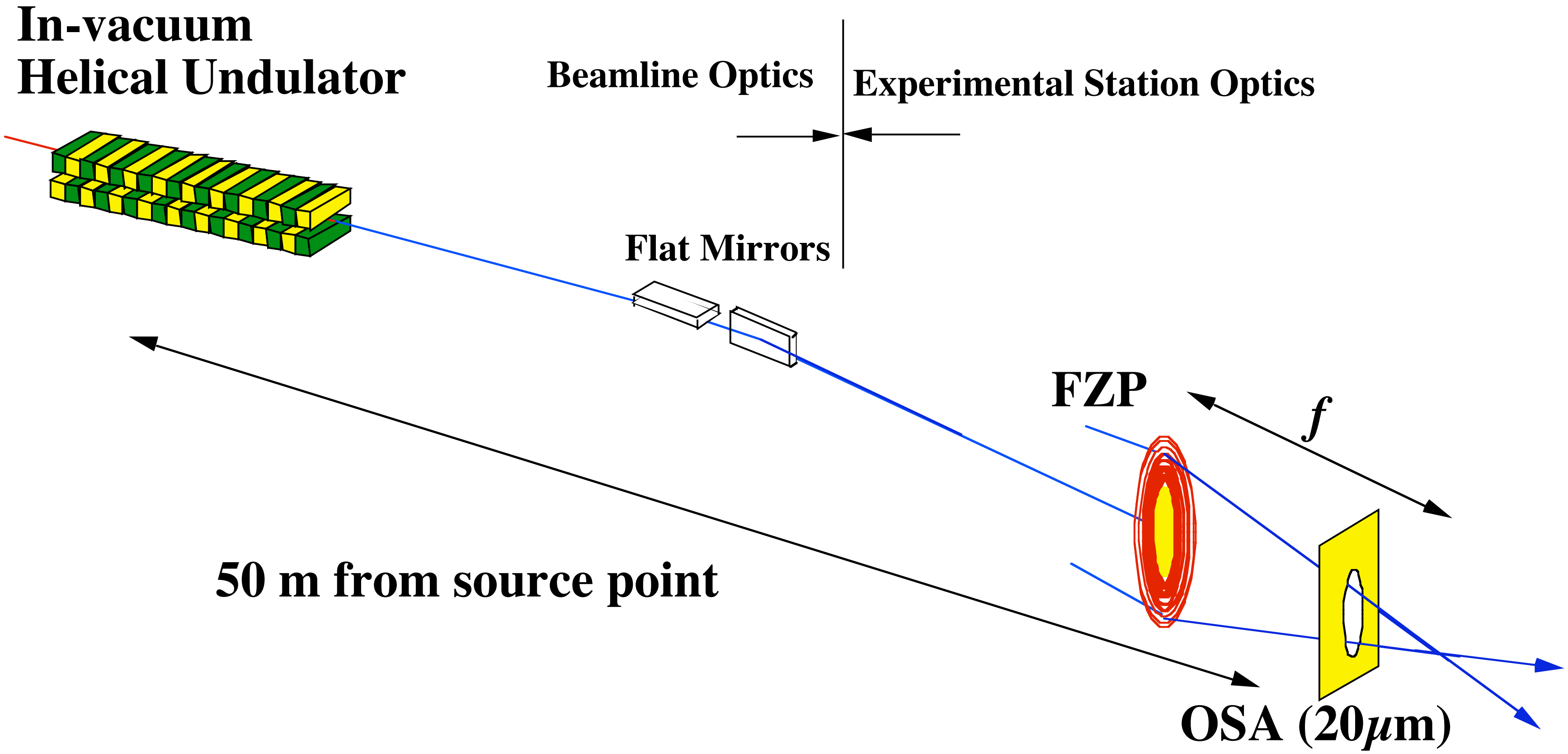
Object: Cu mesh,
2000 lines/inch

Object: Fresnel zone plate,
0.25 μm outermost zone width

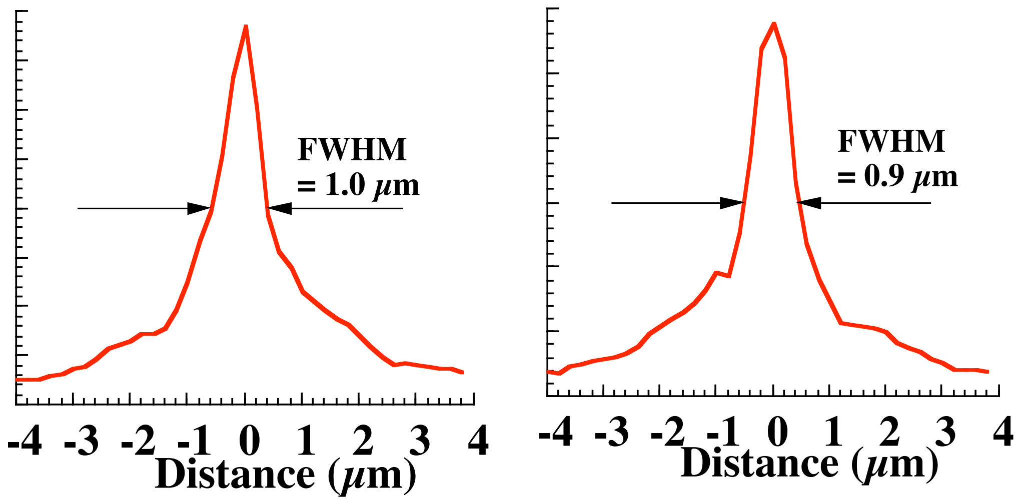
Image of test object

Objective: FZP, 0.25 μm outermost zone width, 100 zones,
Magnification: 11.3,
X-ray energy: 8.34 keV,
Exposure time: 1.5 ms (Single Shot)

Hard X-ray Imaging Microscopy with Fresnel Zone Plate Objective & Quasi-monochromatic Undulator Radiation at BL40XU



Optical Layout of Microbeam Experiment at BL40XU of SPring-8

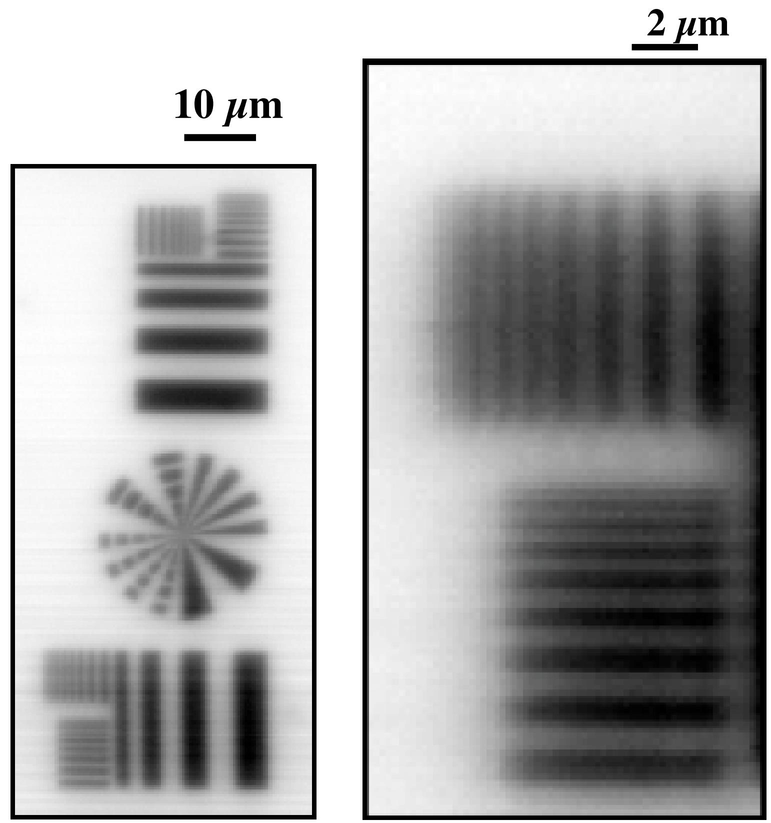


Measured Profiles of Focused Beam

X-ray Energy: 8.317 keV

Total Flux of Focused Beam:

~ 2 x 10^{12} photons/s

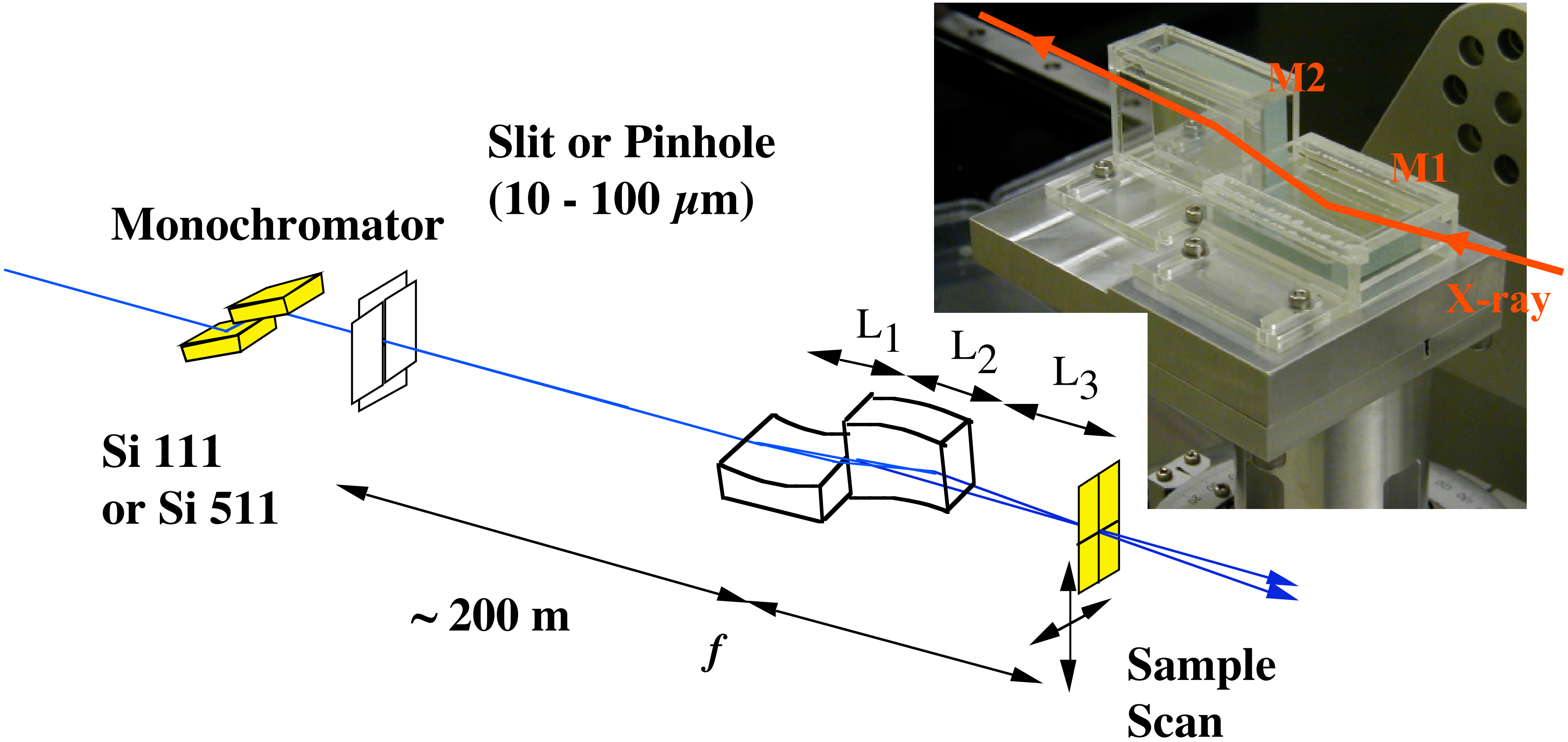


151 x 334 pixels,
0.3 μm/pixel,
0.3 s dwell time.

66 x 126 pixels,
0.2 μm/pixel,
0.2 s dwell time.

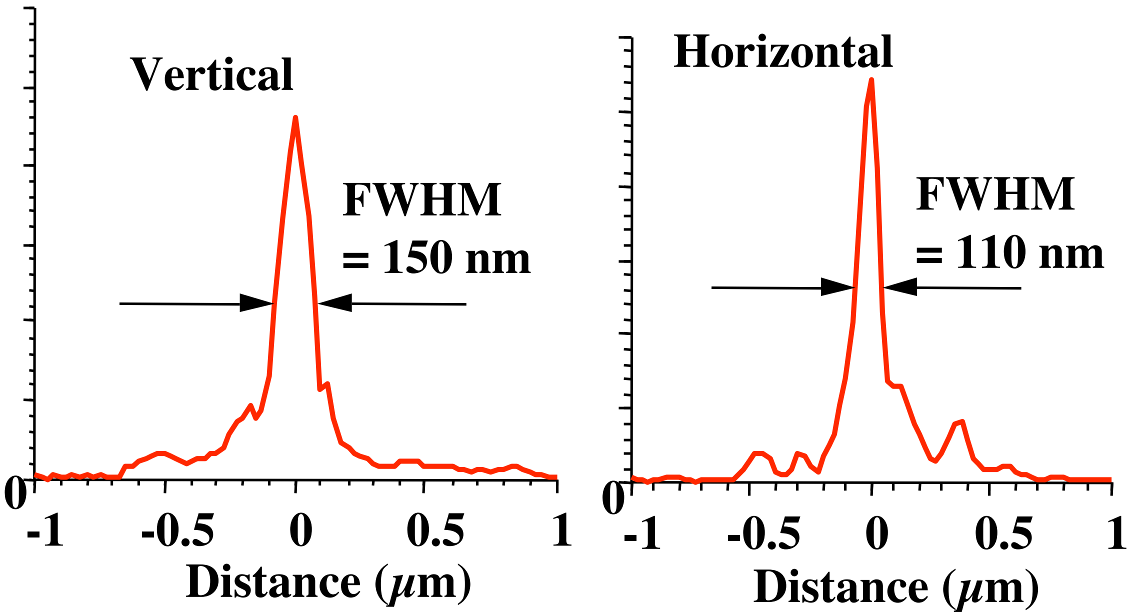
Scanning Microscopic Images of Resolution Test Patterns

Microbeam and Scanning Microscopy with FZP and Quasi-monochromatic Undulator Radiation

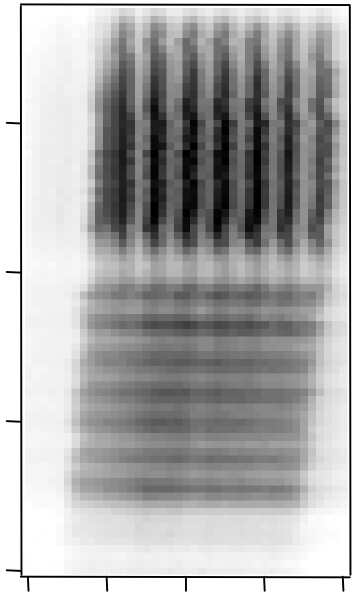


Experimental Setup of X-ray Microbeam/Scanning Microscopy with Total-reflection Mirror Optics (Kirkpatrick-Baez Configuration)

Kirkpatrick-Baez Optics with Aspherical (Plane Parabola) Mirrors,
L1: 45 mm, L2: 45 mm, L3: 25 mm, *f*: 75 mm,
Glancing angle: 2.8 mrad. (Pt coated SiO_2),
Fabricated at Cannon Co. Japan.

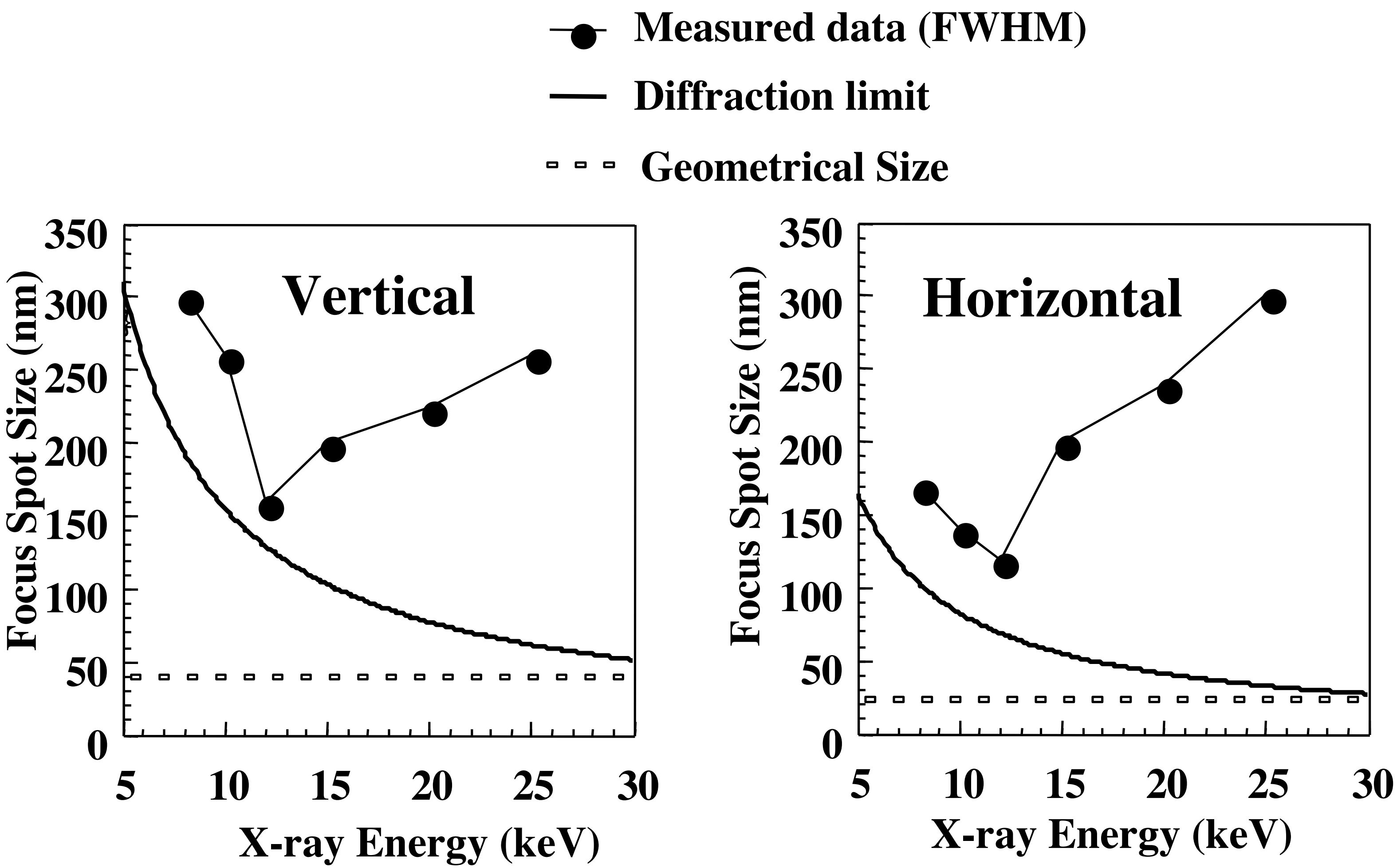


Focused Beam Profiles
measured by Knife-edge Scan
X-ray Energy: 12 keV

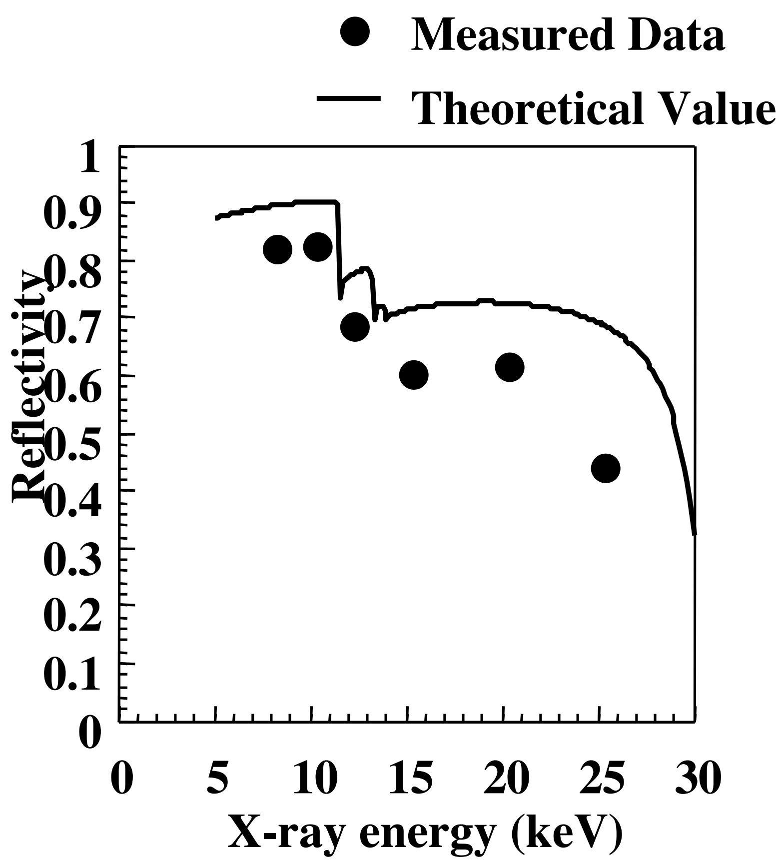


Scanning Microscopy Image
of Test Patterns
0.1 μm line&space

Micirobeam and Scanning Microscopy with Total-reflection Mirror Optics



Energy Dependence of Resolution



Reflectivity of Total Reflection Mirrors

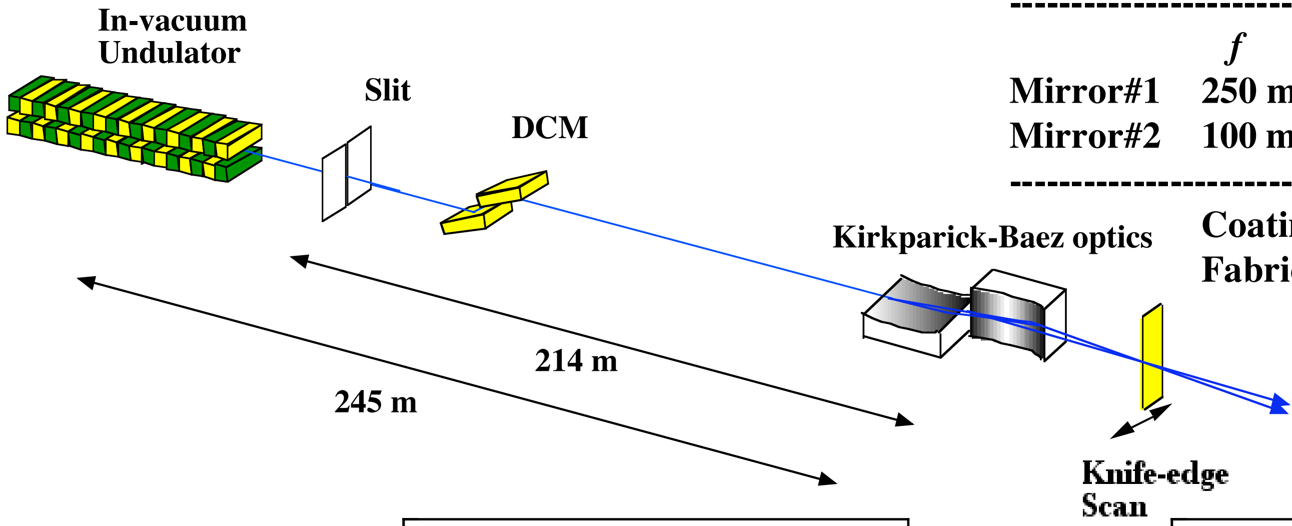
Pt surface,
Glancing angle: 2.8 mrad.

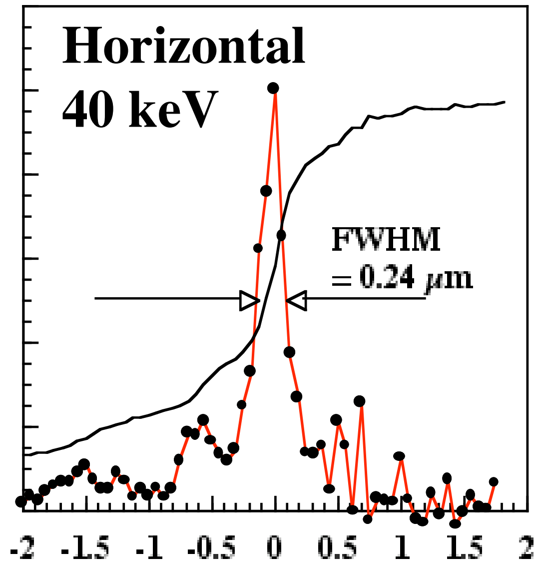
Design Parameters of Parabolic Mirrors

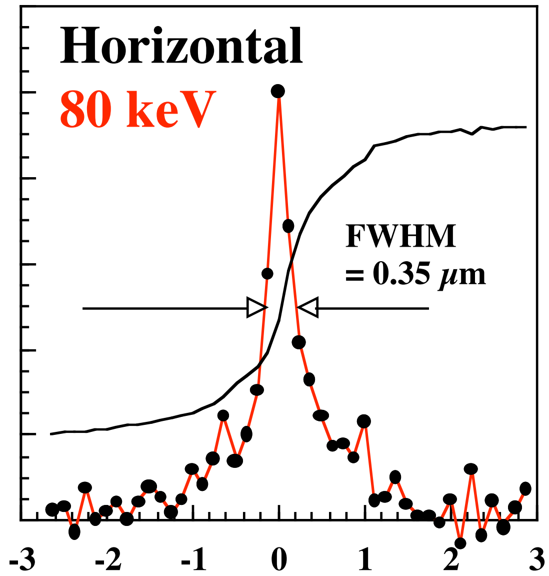
	f	length	Glancing angle	NA
Mirror#1	250 mm	100 mm	0.8 mrad	1.64 x 10^{-4}
Mirror#2	100 mm	100 mm	0.8 mrad	4.78 x 10^{-4}

Coating: Pt

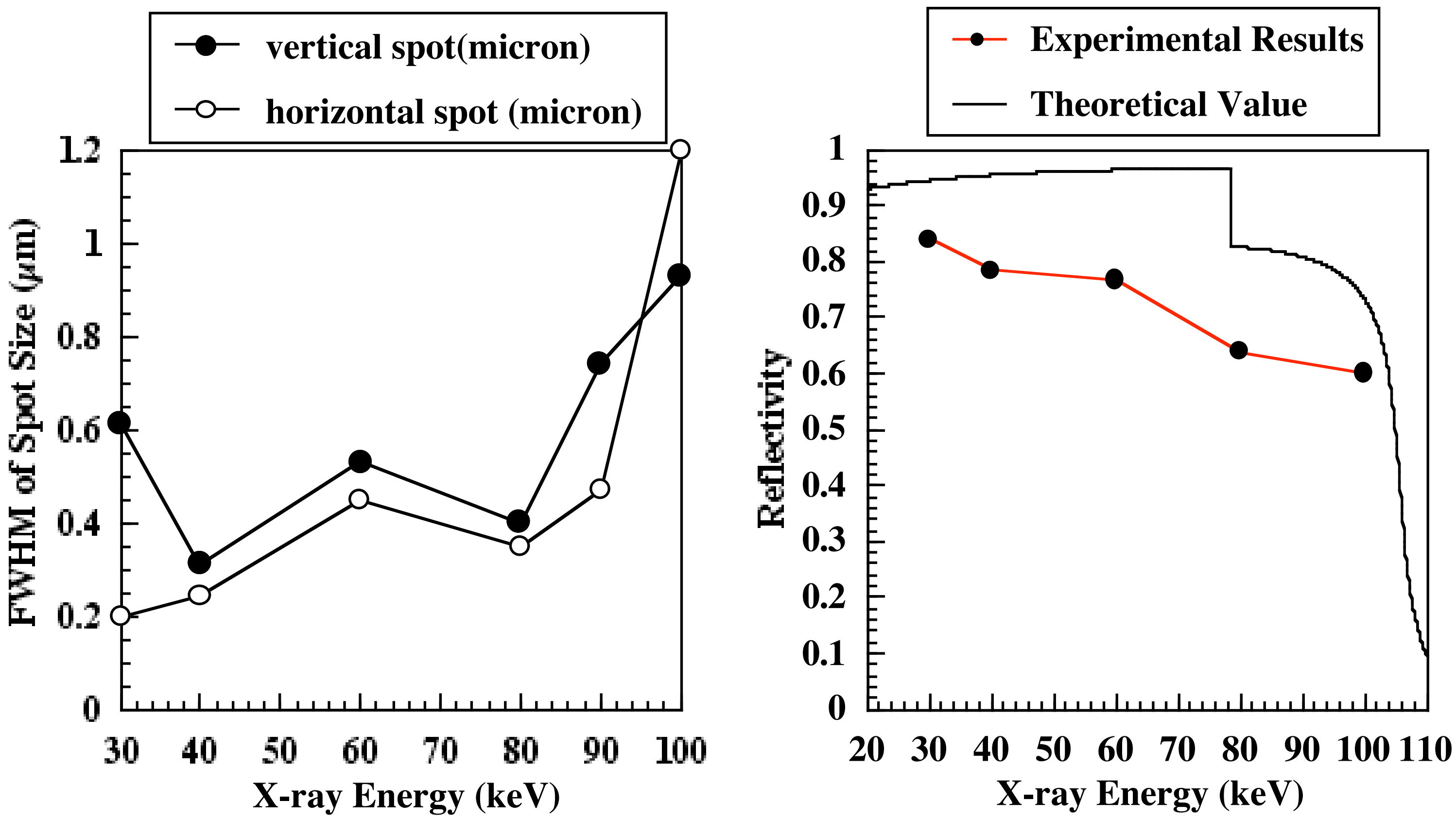
Fabricated at Cannon Co. Japan.



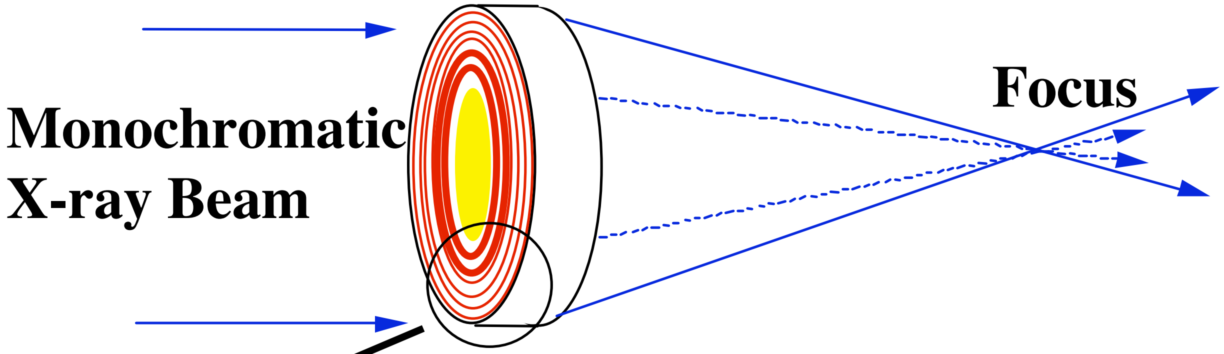


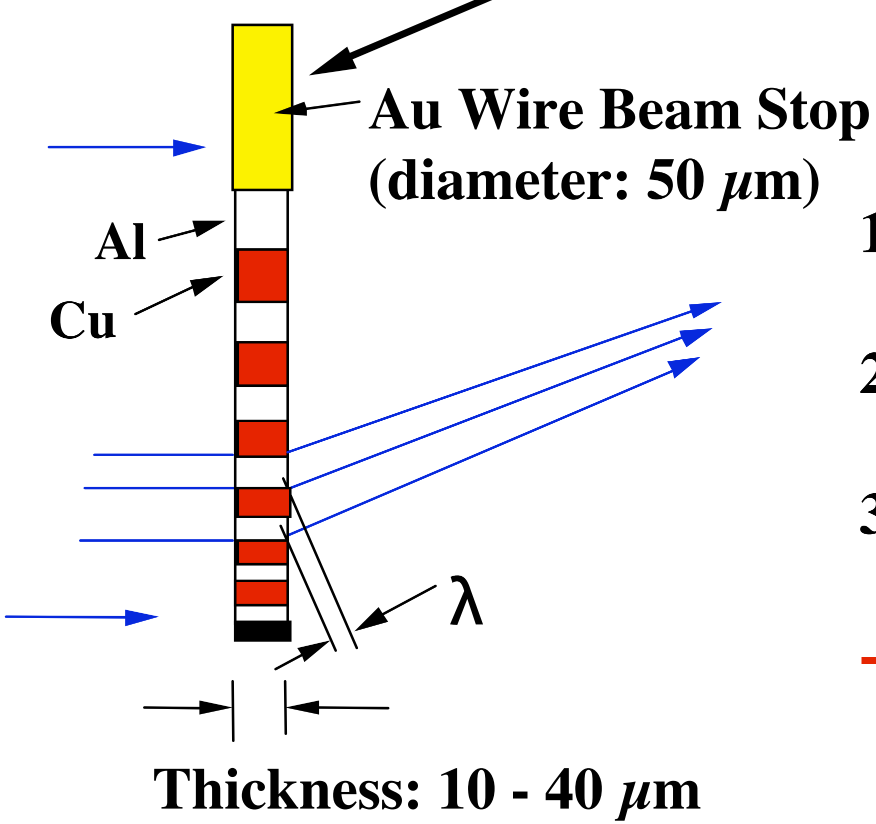


Total Reflection Mirror for High Energy X-ray Microbeam



Total Reflection Mirror for High Energy X-ray Microbeam



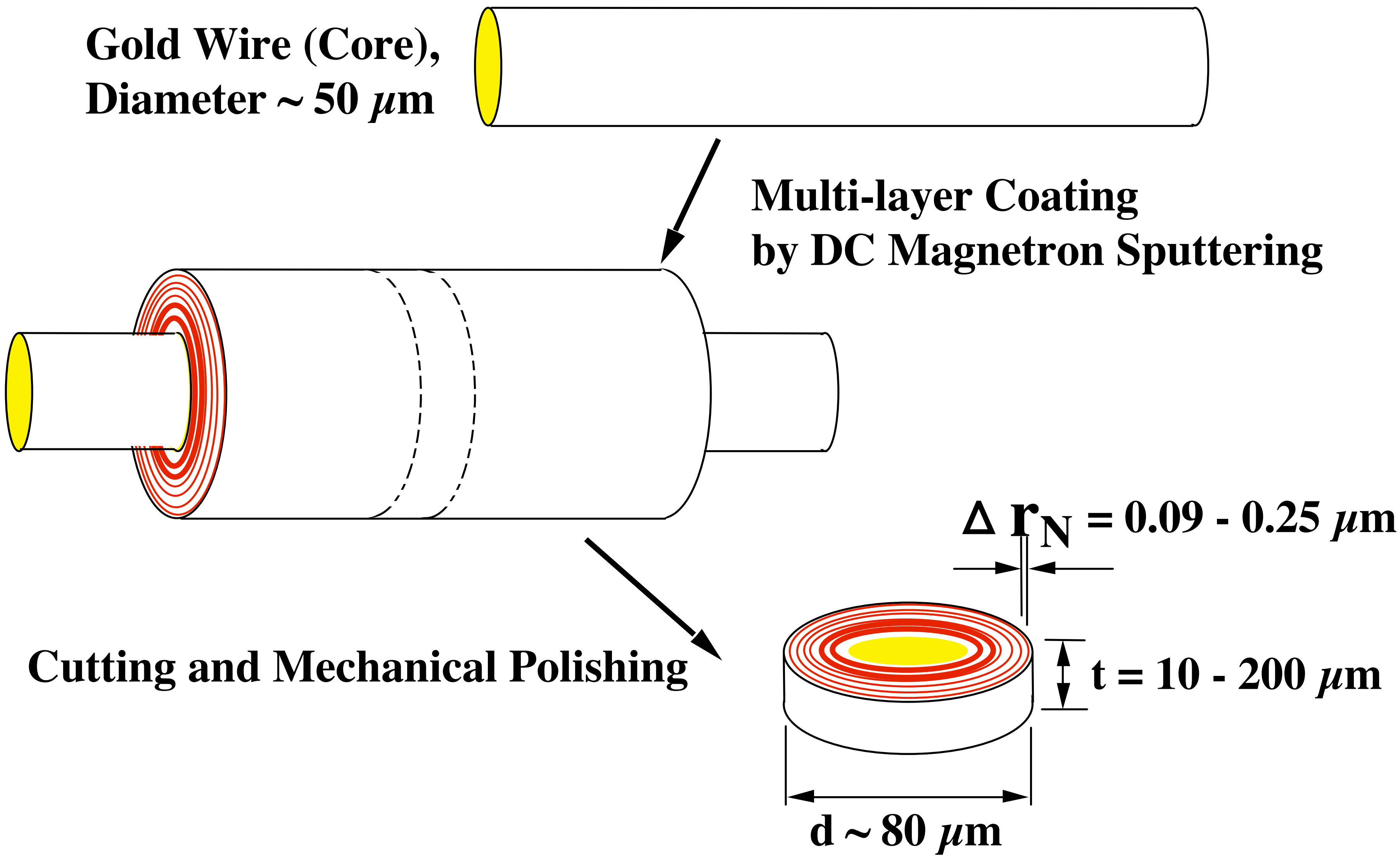


Advantages of SS-FZP

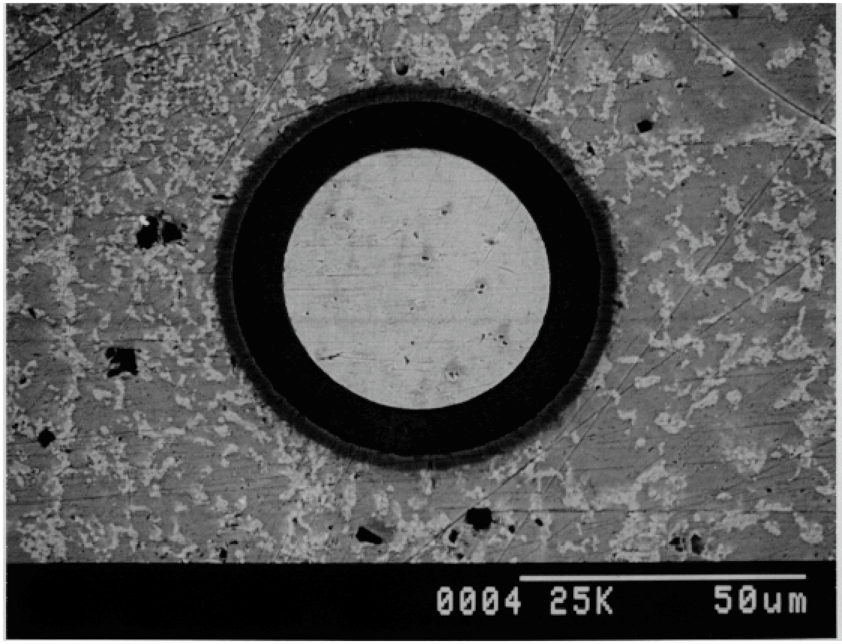
1. **High Aspect Ratio**
   Zone Height/Zone Width > 100.
2. **Narrow Zone Width**
   < 100 nm.
3. **Thickness**
   No Thickness Limit.

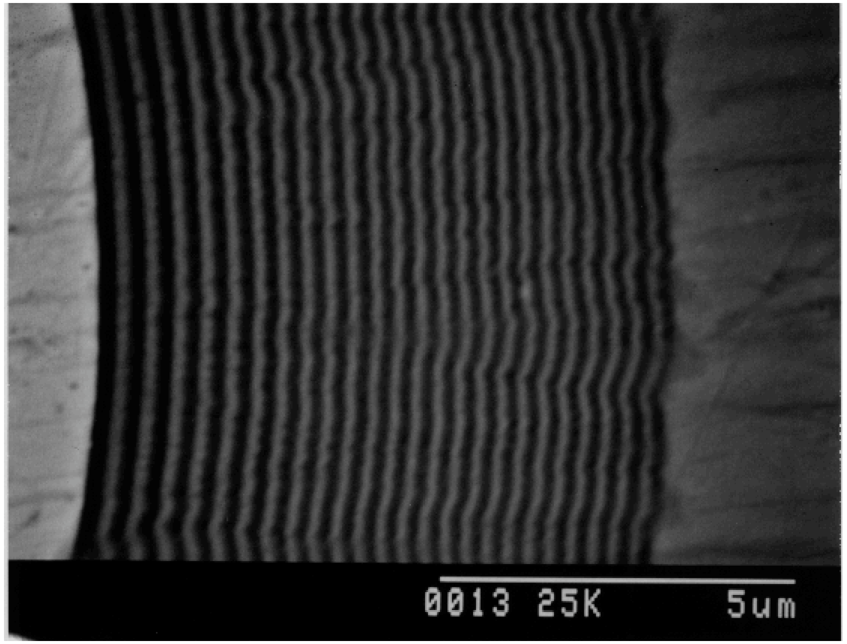
--> High Energy (~ 100 keV) Optics.

Schematic View of Sputtered-sliced Zone Plate



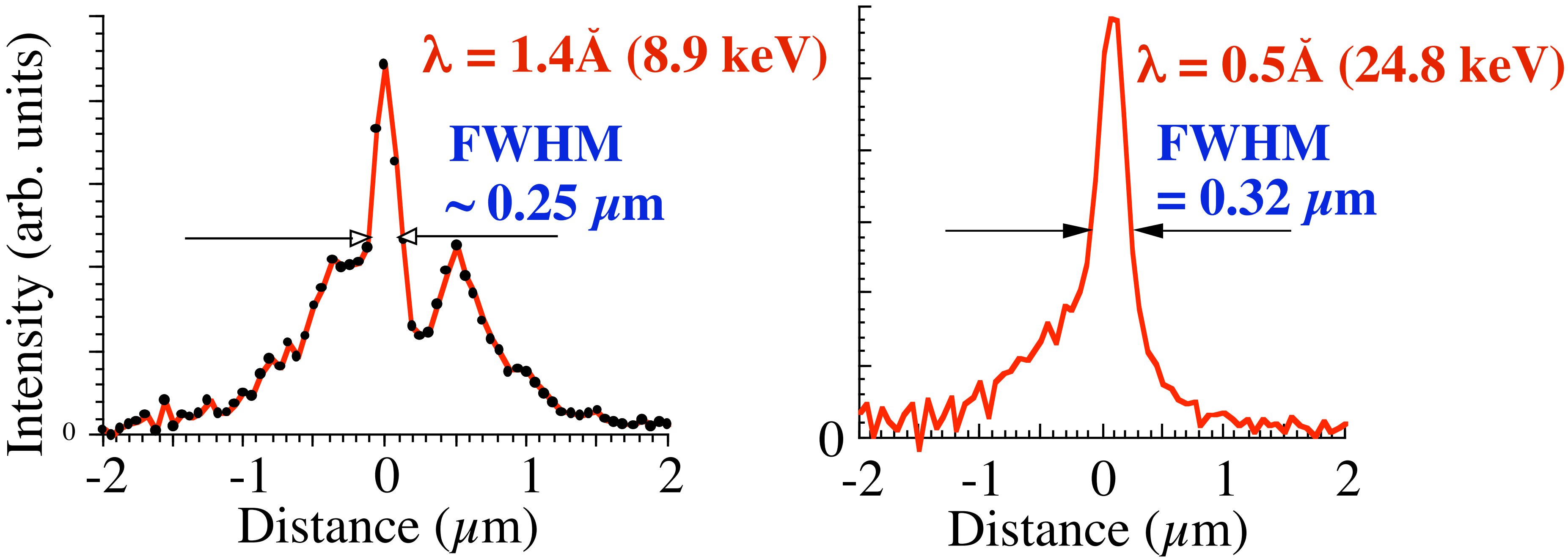
Fabrication Process of Sputtered-sliced Fresnel Zone Plates





SEM Image of Sputtered-sliced Fresnel Zone Plate

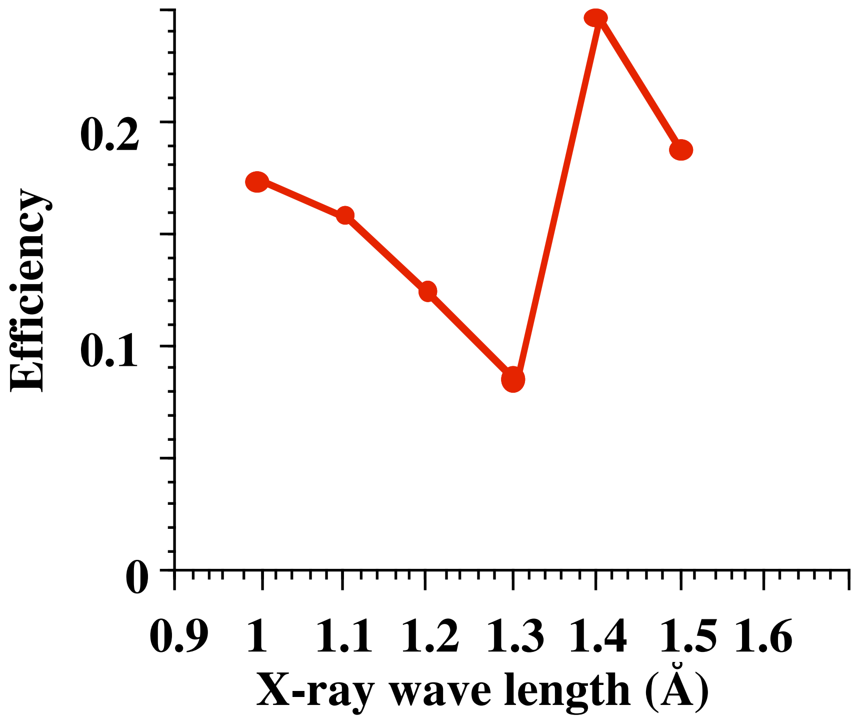
Au Core (50 μm in diameter), Cu/Al 50 Layers,
Outermost zone width of 0.15 μm .



X-ray wavelength: 1.4 Å (8.9 keV),
f ~ 158 mm,
Cu/Al sputtered-sliced FZP (50 layers),
Core (beam stop): Au 50 μm in diameter,
Outermost zone width: 0.25 μm,
Thickness: ~ 20 μm.
Diffraction efficiency: 25% @ 1.4 Å

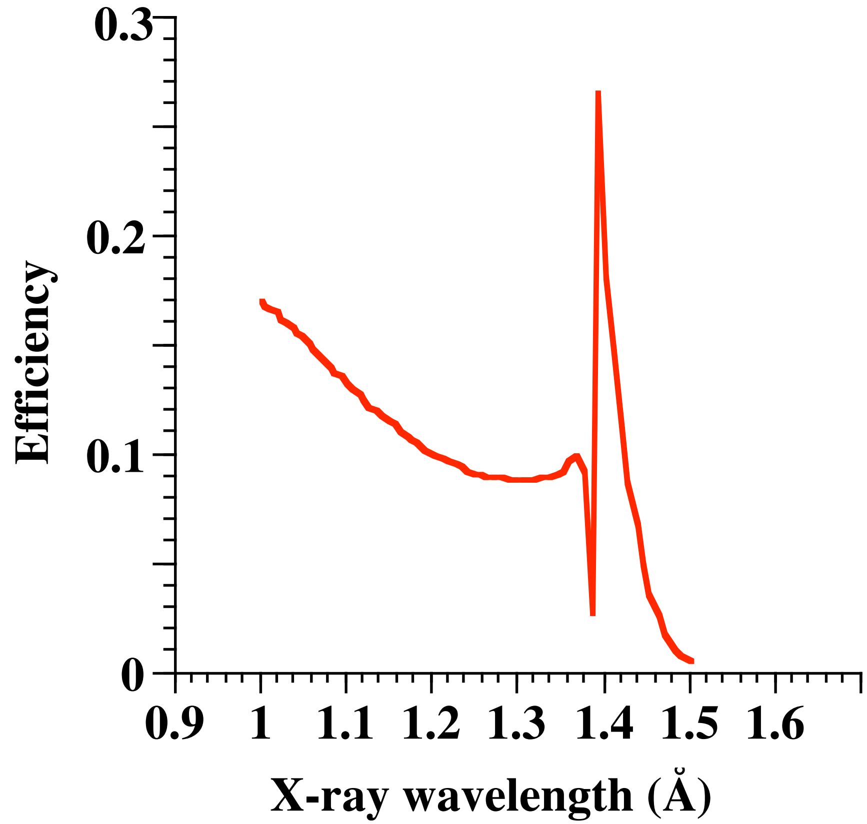
X-ray wavelength: 0.5 Å (24.8 keV),
f ~ 220 mm,
Cu/Al sputtered-sliced FZP (70 layers),
Core (beam stop): Au 100 μm in diameter,
Outermost zone width: 0.09 μm,
Thickness: ~ 60 μm.
Sagittal Focus (1/4 of annular aperture)

Focused Beam Profile Measured by Edge-scan @BL20XU



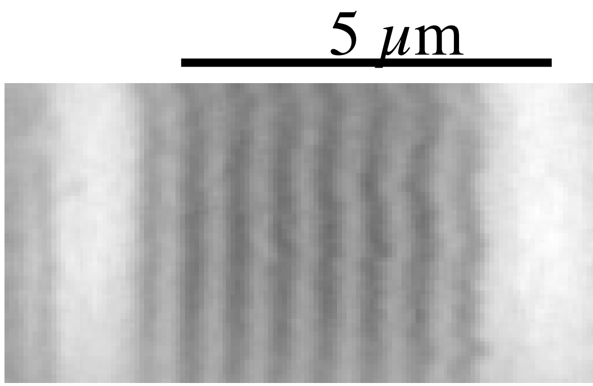
Diffraction efficiency of Fresnel zone plate (first order)

Cu/Al Sputtered-Sliced FZP,
Thickness: ~20 μm.
Core: Gold, 50 μm in diameter,
50 layers.

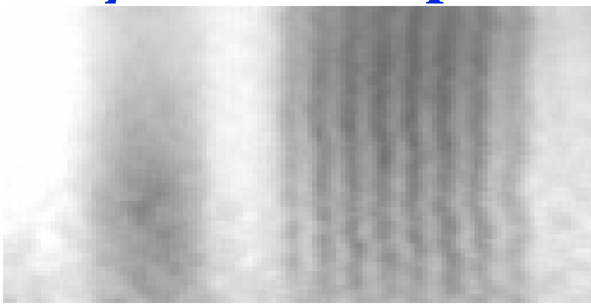


Calculated diffraction efficiency of Fresnel zone plate

Cu/Al multilayer
Thickness: 20 μm

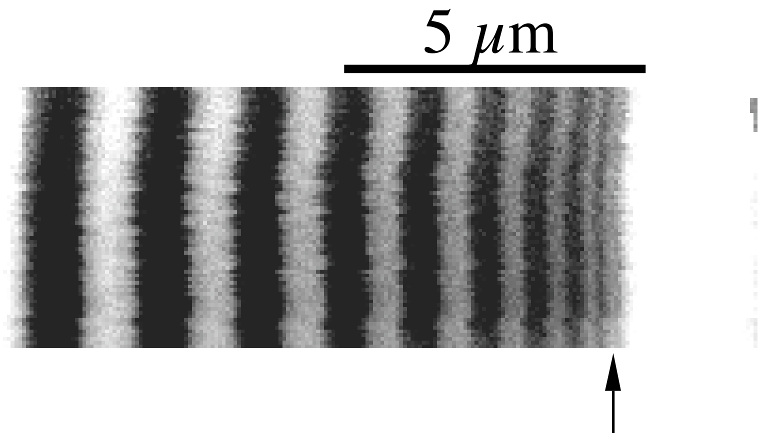


0.3 μm line & space



0.2 μm line & space

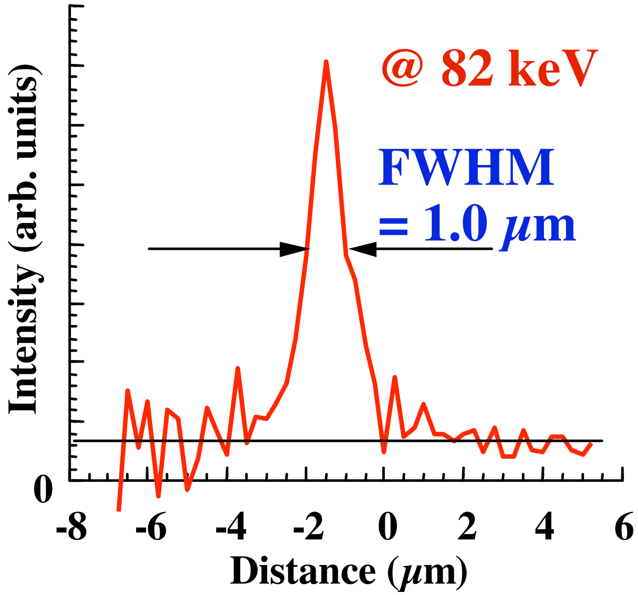
X-ray wavelength: 1.4 Å,
128 x 64 pixels,
0.0625 μm/pixel,
Dwell time: 0.4 s/pixel.



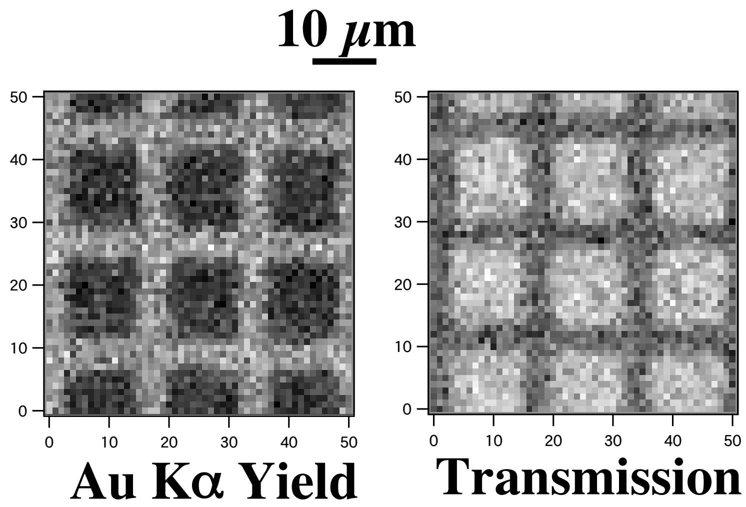
0.1 μm line & space

X-ray wavelength: 1.0 Å,
256 x 70 pixel,
0.0625 μm/pixels,
Dwell time: 0.4 s/pixel.

Scanning Microscopic Image of Resolution Test Pattern



Focused Beam Profiles
measured by Knife-edge Scan



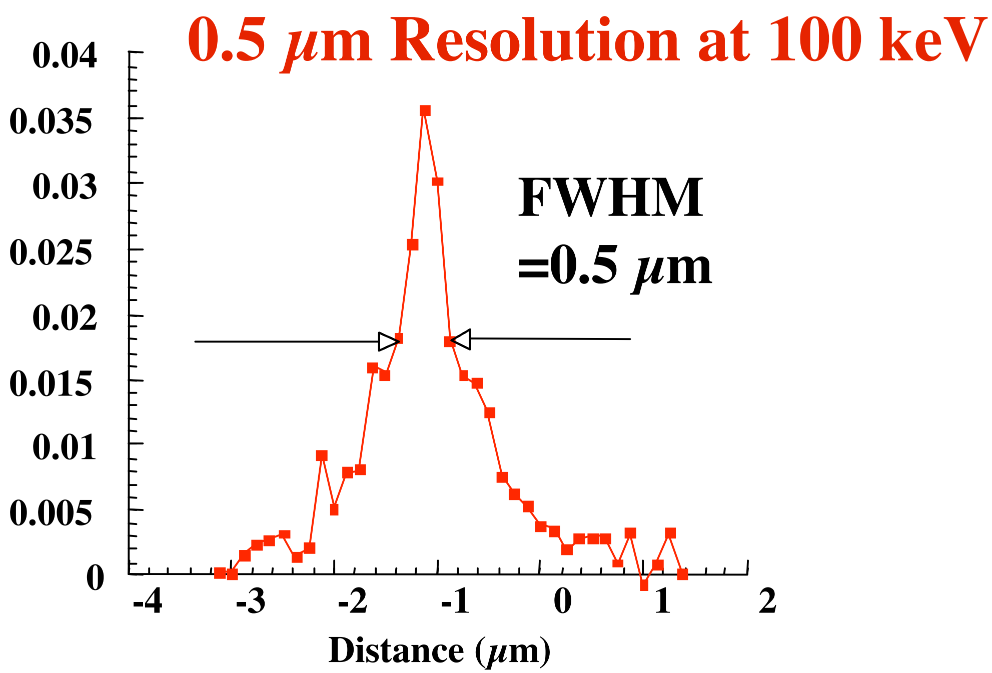
Scanning Microscopic Image

X-ray Energy: 82 keV (0.151 Å), f ~ 700 mm,
Cu/Al sputtered-sliced FZP (50 layers),
Core (center beam stop): Au 50 μm in diameter,
Outermost zone width: 0.15 μm,
Thickness: ~ 40 μm.

Sample: gold mesh (1500 lines/inch),
X-ray Energy: 82 keV ,
51 x 51 pixels,
1 μm/pixel,
Dwell time: 2 s/pixel,
CdZnTe-detector for fluorescent X-rays.

Microfocusing/scanning microscopy with SS-FZP at 82 keV

Diffraction efficiency: 15%



Microbeam with Sputtered-sliced FZP

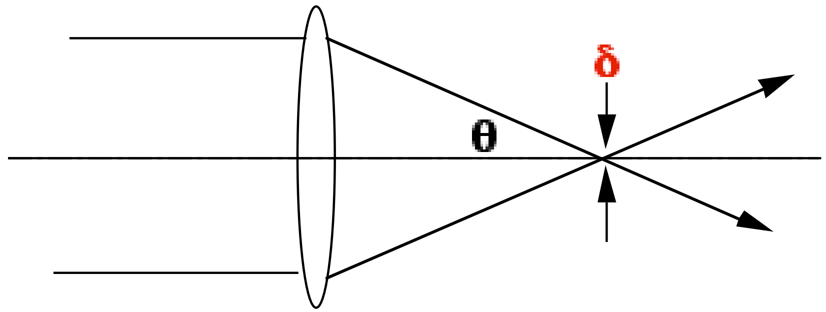
Focused Beam Profile Measured by Edge-scan @BL20XU

X-ray wavelength: 0.124 Å (100 keV), f ~ 900 mm,
Cu/Al sputtered-sliced FZP (70 layers),
Core (beam stop): Au 50 μm in diameter,
Outermost zone width: 0.16 μm,
Thickness: ~ 180 μm.

Resolution Limit of X-ray Microscope

General Theory

Rayleigh's criterion (Diffraction Limit in Classical Optics)



$\delta = C \lambda / NA$,

$NA = n \sin\theta$,

n: Index of Refraction,

$C \sim 1$ (constant, dependent on optics configuration).

Typically, $C \sim 0.61$ (Circular aperture), $n \sim 1$ (in air), $\sin\theta \sim 0.5$ ($F \sim 1$) for visible light,

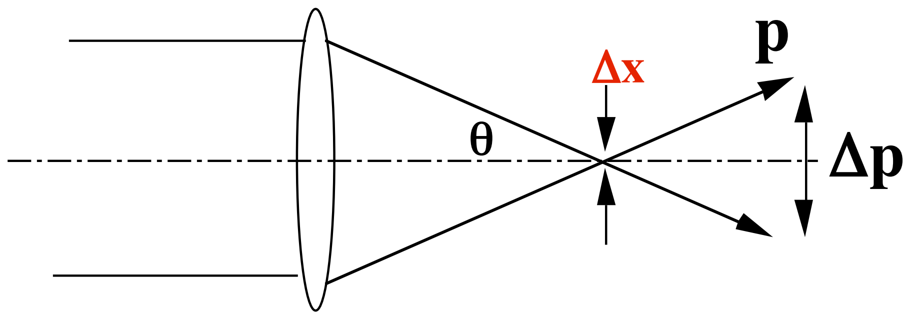
$\delta \sim \lambda$: Resolution limit of Optical Microscope.

General Theory

Uncertainty Principle (Quantum Mechanics)

$$\Delta p \Delta x \geq h$$

Momentum of Photon: h/λ,
Momentum Spread by Focusing Optics: $\Delta p = 2|p| \sin\theta$.

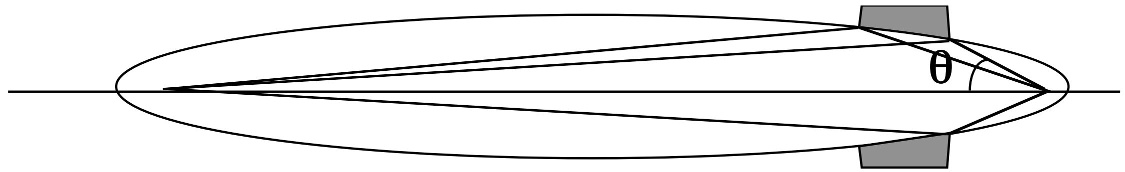


$$\Delta x \geq \lambda/(2\sin\theta)$$

How about hard X-ray microscopy?

Total Reflection Mirror Optics

Elliptical Mirror Optics:



$$NA_{max} = 2\theta_c$$

$$\Delta x \geq 0.61 \times \lambda/(2\theta_c). \qquad (2)$$

Using free-electron approximation,

$$\theta_c(rad) \sim 1.6 \times 10^{-2}\, \lambda \rho^{1/2}, \qquad (3)$$

ρ (g/cm3): Density of Mirror Material,
λ (nm): X-ray wavelength.

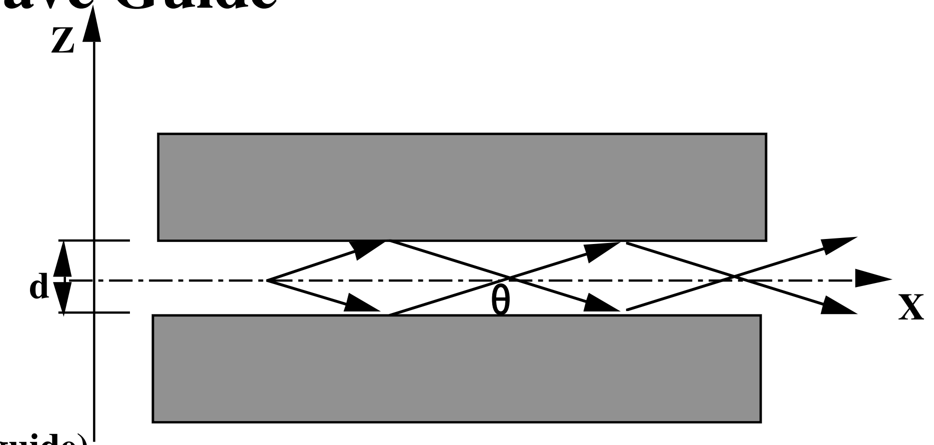
The theoretical limit of spatial resolution, Δx, is determined only by the density of the reflector surface material, $\sqrt{\rho}$.
The limit of spatial resolution is approximately 10 nm.

For combined mirror optics
(Wolter-type-mirror or tandem-toroidal-mirror optics),

$$\Delta x = 0.61 \times \lambda/(4\theta_c). \qquad (4)$$

1. P. Kirkpatrick and A. V. Baez: J. Opt. Soc. Am. **38** (1948) 766.
2. Von H. Wolter: Ann. Physik **10** (1952) 94.
3. Y. Sakayanagi: Optica Acta **23** (1976) 217.

X-ray Wave Guide



Planar wave guide, 1-D solution,

Boundary Condition: $2d \sin\theta = m\lambda$, m = 1, 2, 3, ...

Lowest mode of propagating wave: m = 1,

$d \sin\theta = \lambda/2$,

d: gap of waveguide (inner diameter of waveguide)
θ: glancing angle to wall

$\theta \leq \theta_c$ (θ_c : critical angle for total reflection)

When the phase jump at total reflection = π (case of $\theta << \theta_c$), minimum size of wave guide, d_o,

$d_o = \lambda/(2\theta) \leq \lambda/(2\theta_c)$.

However, the penetration depth of evanescent wave, t,

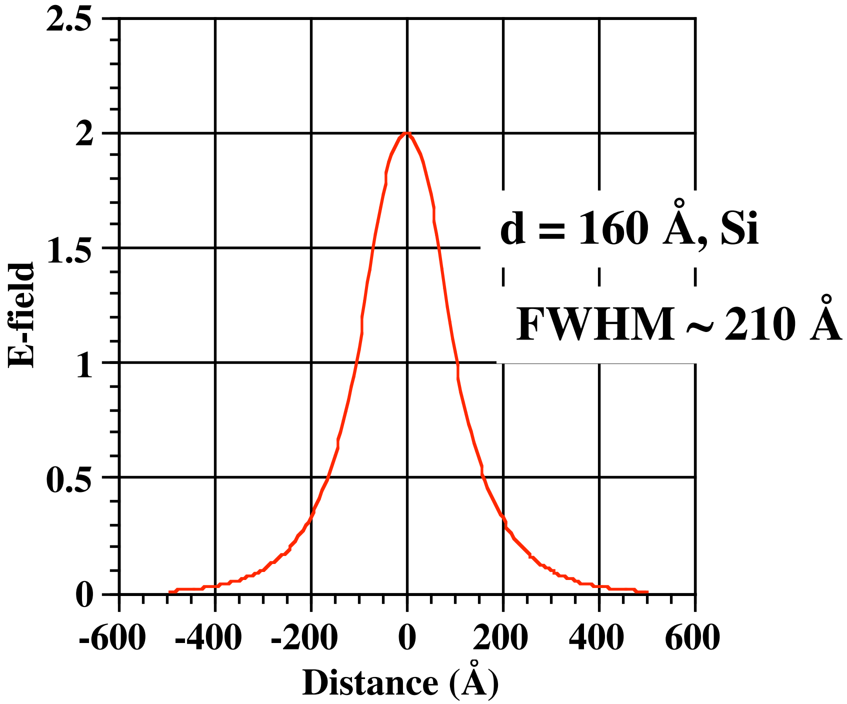
$t \sim \lambda/(\theta_c^2 - \theta^2)^{1/2}$. So, effective broadening of wavepacket is

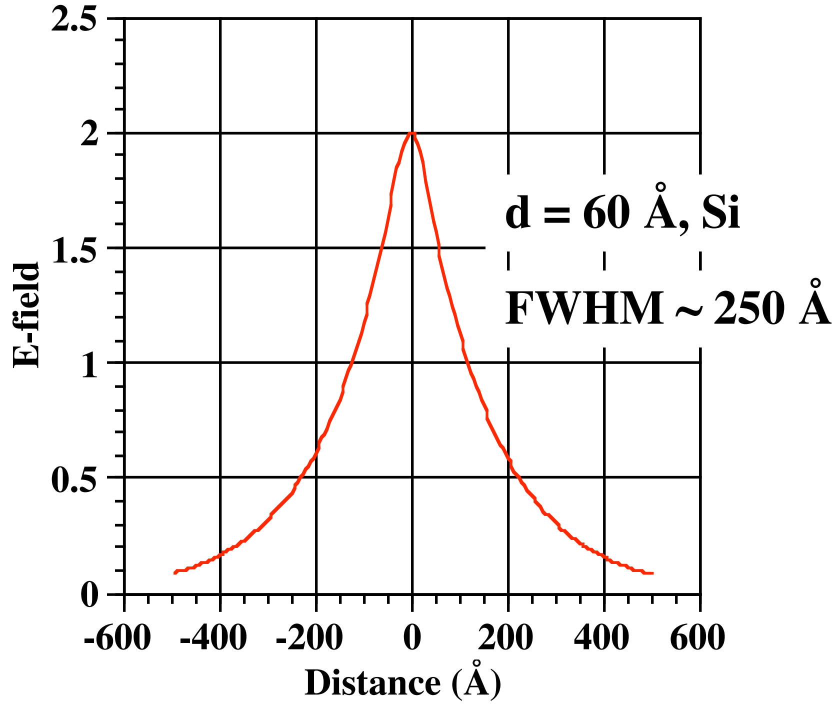
$\Delta x \sim t \sim \lambda/\theta_c$: the same as that of total reflection mirror optics.

or $\Delta x \leq \lambda/(2\theta_c)$, simply from uncertainty principle.

cf. C. Bergemann, H. Keymeulen and J. F. van der Veen: Phys. Rev. Lett. **91** (2003) 204801.

Numerical Calculation of Wave-packet in Wave-guide

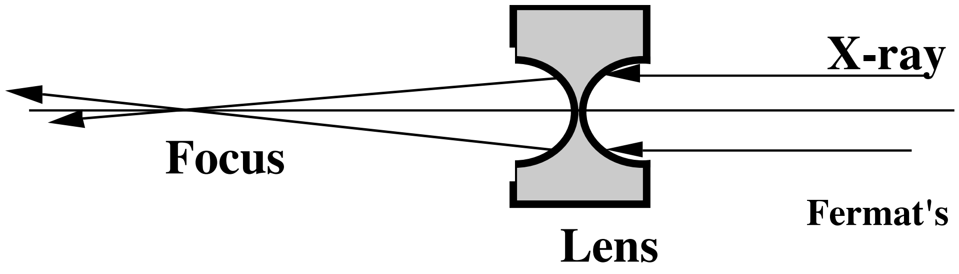




Electric Field Intensity in the Wave-guide

Si (ρ = 2.34), λ = 1.28Å,
d = 60 Å, and d = 160 Å.

Refractive Lens Optics



Spherical Lens: Spherical Aberration

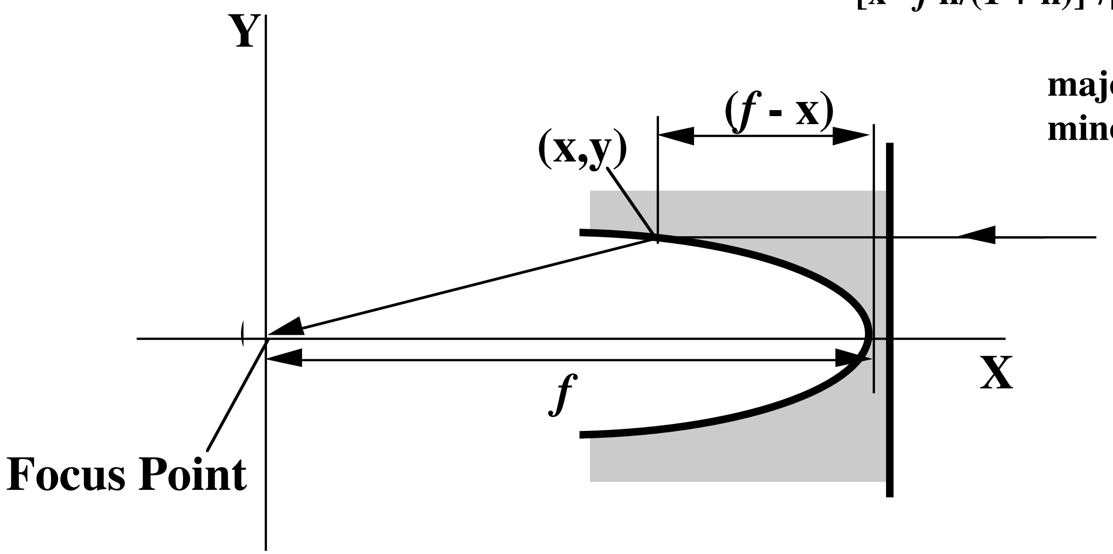
Fermat's principle of least time.

$$(x^2 + y^2)^{1/2} + n(f - x) = f, \qquad (6)$$

n: index for refraction

$$[x - f\, n/(1 + n)]^2/[f^2/(1 + n)^2] + y^2/[f^2(1 - n)/(1 + n)] = 1\,, \qquad (7)$$

major axis of ellipse: $f/(1 + n)$,
minor axis of ellipse: $f\,[(1 - n)/(1 + n)]^{1/2}$.



Refractive Lens: Exact Solution

Diffraction-limited Resolution of Single Refractive Lens

Considering phase shit of 2mπ, (mλ, m = 1, 2, 3, ...)

$$(x^2 + y^2)^{1/2} + n(f - x) = f + m\lambda, \quad (8)$$

$$[x - \{f + m\lambda/(1 - n)\}n/(1 + n)]^2/[\{f + m\lambda/(1 - n)\}^2/(1 + n)^2] + y^2/[\{f + m\lambda/(1 - n)\}^2(1 - n)/(1 + n)] = 1. \quad (9)$$

Numerical Aperture of the Lens (NA):

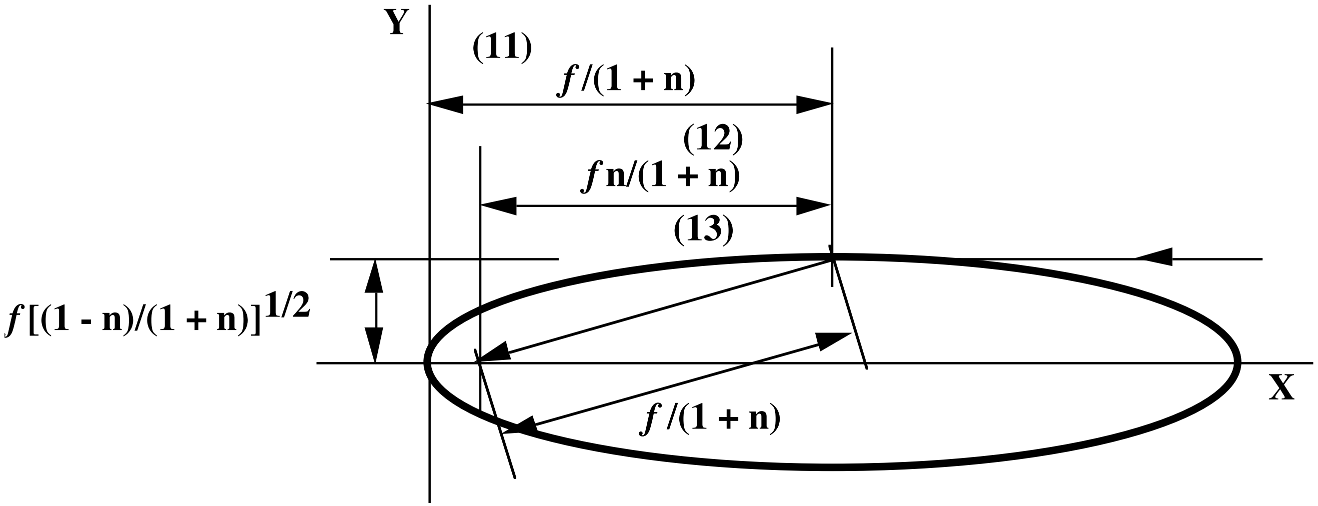
$$NAmax = [(1 - n)/(1 + n)]^{1/2}/[1/(1 + n)] = (1 - n^2)^{1/2}. \quad (10)$$

Using n = 1 - δ, and δ << 1,

$$NAmax \sim (2\delta)^{1/2}.$$

$$\theta c \sim (2\delta)^{1/2}.$$

$$\Delta = 0.61 \times \lambda/\theta c.$$



cf. Y. Suzuki, Jpn. J. Appl. Phys. 43 (2004) 7311-7314.

Expansion to Fresnel Lens and Fresnel Zone Plate

The nesting configuration, the series of ellipsoids m = 0, 1, 2, 3, , , , M
Fresnel zone plate at $f = x$:

$$(f^2 + y^2)^{1/2} = f + m\lambda. \quad (14)$$

$$y = [2m\lambda f + (m\lambda)^2]^{1/2}. \quad (15)$$

When $f >> m\lambda$, by neglecting the higher-order terms,

$$y = (2m\lambda f)^{1/2}. \quad \text{[Zone Plate Equation]} \quad (16)$$

The major axis of the ellipse: $\{f + m\lambda/(1 - n)\}/(1 + n)$,
The major axis of the ellipsoid for the outermost zone should be smaller than the focal length f.

$$\{f + m\lambda/(1 - n)\}/(1 + n) \le f. \quad (17)$$

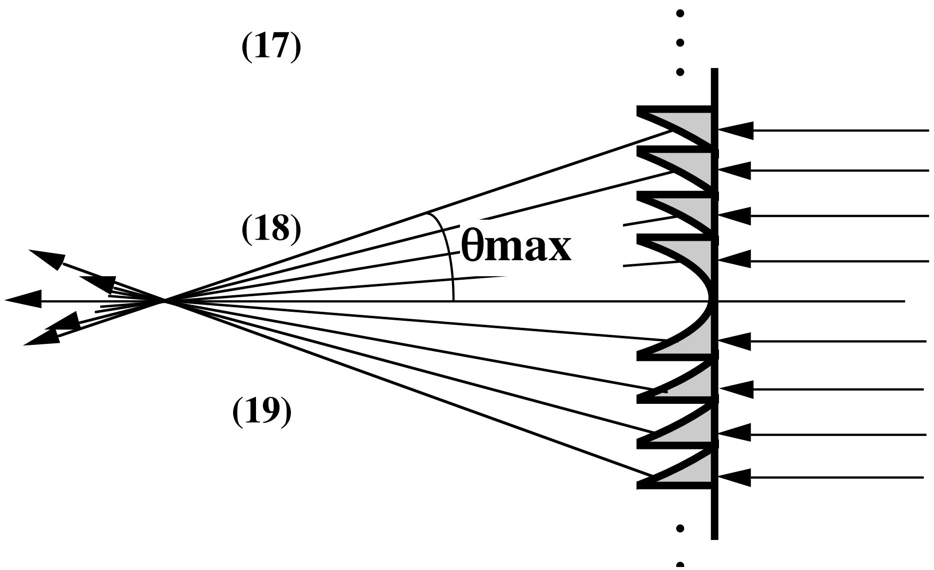
The possible outermost zone for the planar zone plate:

$$\{f + M\lambda/(1 - n)\}/(1 + n) = f. \quad (18)$$

M: the maximum m.

$$M\lambda/(1 - n) = n f. \quad (19)$$

$$\theta max = (2M\lambda f)^{1/2}/f = [2f^2 n(1 - n)]^{1/2}/f \sim (2\delta)^{1/2},$$



cf. Y. Suzuki, Jpn. J. Appl. Phys. 43 (2004) 7311-7314.

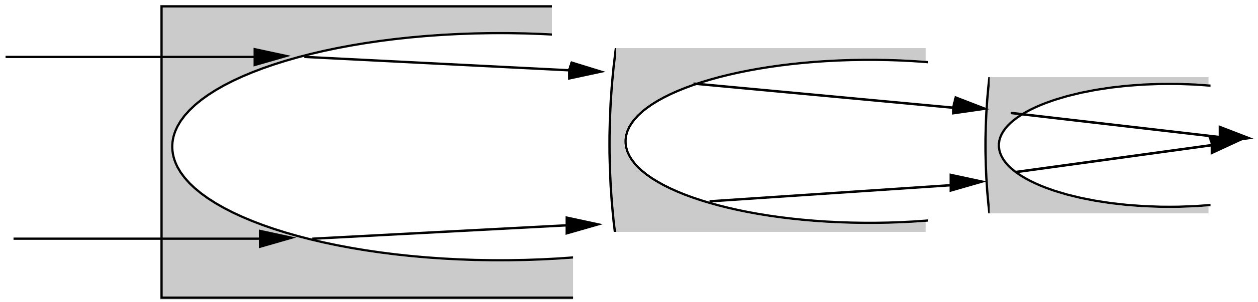
Theoretical Resolution Limit of

Total Reflection Mirror Optics,
Wave-Guide,
Refractive Lens,
Fresnel Zone Plate

$\sqrt{(2\delta)} \sim 10$ nm in hard X-ray Region

Possible Ways to Nanometer Resolution

1. Combined Refractive Lens,



Diffraction Limited Resolution

$\sim 0.61 \times \lambda/(N\theta c)$

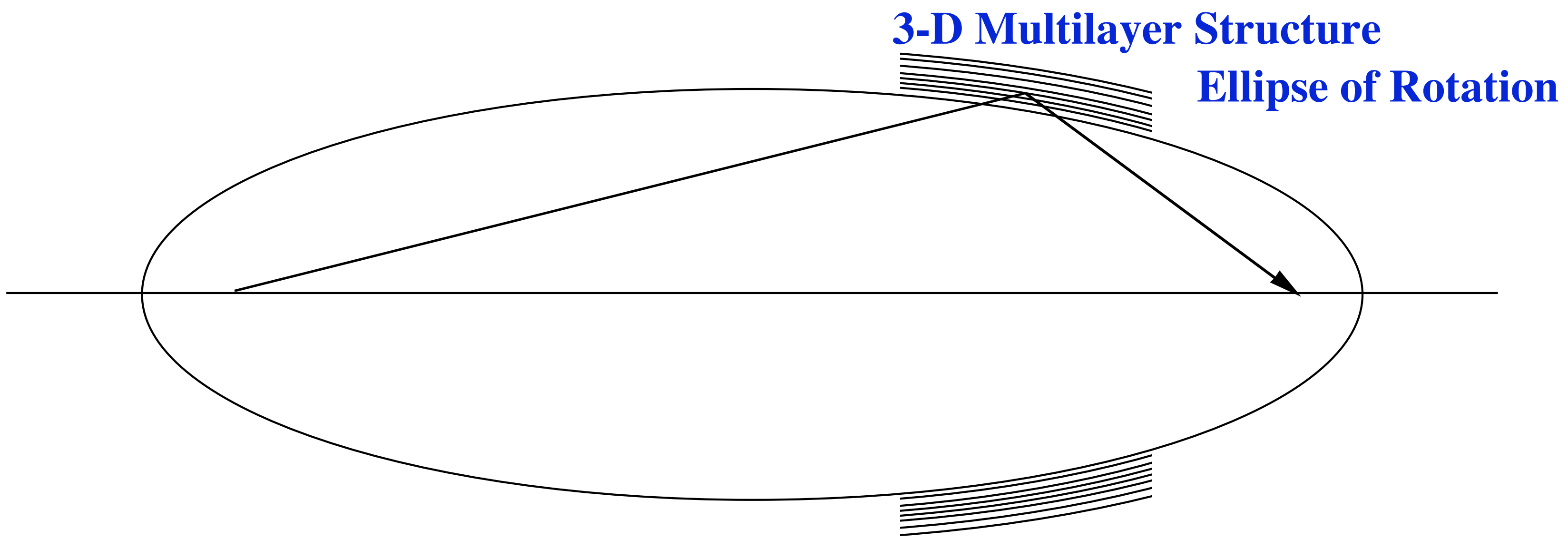
N: Number of Combined Lens

Spherical lens might be feasible,
because smaller lens has smaller aberration.

cf. C. Schroer and B. Lengeler, Phys. Rev. Let. 94 (2005) 054802

Possible Ways to Nanometer Resolution

2. Three Dimensional Zone Plate
(Volume Zone Plate or Laue Lens)



Nested Multilayer Structure of Ellipse of Rotation with Optical Path Difference of mλ.

H. C. Kang et al., Phys. Rev. Lett. 96 (2006) 127401,
C. Schroer, Phys. Rev. B 74 (2006) 033405.

Ideal only on Focusing Property.

Next limit: atom size with λ/4 rule (Rayleigh limit)
~ 1 nm.

ERL & FEL

Complementary?

Which is better for users?

Time structure & spectral structure.

Nano-optics:

Applications? Users? Practical?

R&D of optics: 10 nm resolution -> 1 nm resolution..?

Problems in the 3rd-generation SR source, Spring-8

Most of users and experiments are 2nd generation!

Important Problems in Coherent X-ray Sources

1. Vibration: optics, light source, ground&building

2. Temperature stability: ~0.01^{o} environment.

3. Radiation damages, cooling.
   Same as Spring-8?

4. Speckles:
   No optics without any speckles.

5. No optics is best optics?